The Biometrics Section of the American Statistical Association

BIOMETRICS

Table of Contents

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AN OPEN LETTER TO BIOMETRICS' SUBSCRIBERS

Because of the current rise in costs, the Board of the American Statistical Association has found it necessary to restrict the number of pages of its journals for the coming year in order to keep expenses within the amount of funds currently available. For Biometrics, a limit of 64 pages per issue has been recommended by the Board subject to final approval by the Council plus any additional pages purchased by the Biometric Society.

Before this decision was reached the editors of Biometrics had planned on approximately 80 pages for each of the four issues during 1949. Contributors have supplied enough valuable papers to make this easily possible. When the above decision by the Board became known, a number of individual subscribers to Biometrics, feeling that reduction of the size of the journal at this time would be undesirable, banded together informally in an effort to permit continued growth in the size of Biometrics. The Association quoted a price of $20.00 a page to this group, this being the price charged the Biometric Society.

Funds contributed immediately were sufficient to increase the present issue to 98 pages. Funds for additional pages in the next three issues are solicited. If you agree that the publication of valuable papers such as have been appearing in Biometrics should be facilitated rather than restricted, please send your contribution at once to the editor:

Miss Gertrude Cox, Editor
c/o Institute of Statistics
State College Station
Raleigh, North Carolina

In this regard, the following information may be of interest. It has been suggested by the Board of the American Statistical Association that the Institute of Statistics, Inc. at North Carolina State College, Raleigh, North Carolina be requested to publish Biometrics. Negotiations are in progress between the Association and the Institute to make this change following publication of the present (March 1949) issue. If this move is completed, the cost for additional pages may be somewhat lower than the $20.00 quoted above. In any case the price will not be more than $20.00 a page, and all money contributed for this purpose will be used to expand Biometrics.

For "Friends of *Biometrics*"
J. Berkson
M. C. Bruyere
J. W. Tukey
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TRIPLE RECTANGULAR LATTICES

Boyd Harshbarger

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The theory of the experimental arrangement which is now called the lattice as given by Cochran [1] and Yates [2,3,4] requires that the number of varieties be an exact square. To avoid the restriction that the number of varieties or treatments be a perfect square, Yates [2] introduced the design which he called pseudo-factorial with unequal groups of sets. No attempt was made to use the recovery of information, and the design proposed by him does not conveniently lend itself to such an analysis.

This paper and a previous one, *Rectangular Lattices*, [5] by the author, present an extension of the incomplete block designs to the cases where the number of varieties or treatments are expressible as the product of two integers. An explicit solution is given for the cases where the number of varieties are two consecutive integers. This paper treats, in particular, the cases where there are three groups of sets. These arrangements differ from Yates' non-square design since the blocks are all the same size and the varieties are adjusted by both the inter- and intra-block information. The name triple rectangular lattices is proposed for the design where there are three groups of sets.

As in the square triple lattices, the varieties for the triple rectangular lattices are arranged in three groups, X. Y. and Z, each of which is replicated as shown below. For simplicity of illustration, a 4×3 triple rectangular lattice is used. The numbers designate varieties.

In practice the varieties are randomized within blocks and the blocks are randomized within the replicates.

For purposes of enumeration and computation triple rectangular lattices may be thought of as square lattices with k varieties missing in the main diagonal. With this arrangement the groups for a 4×3 triple rectangular lattice (together with more convenient subscripts) are shown in Table II.

Group Y is formed from group X by using the columns as blocks; group Z is formed by superimposing the Latin square of Table III upon group X.

TABLE I

GROUP X

Blocks				Blocks			
(1)	1	2	3	(1)	1	2	3
(2)	4	5	6	(2)	4	5	6
(3)	7	8	9	(3)	7	8	9
(4)	10	11	12	(4)	10	11	12

GROUP Y

Blocks				Blocks			
(1)	4	7	10	(1)	4	7	10
(2)	1	8	11	(2)	1	8	11
(3)	2	5	12	(3)	2	5	12
(4)	3	6	9	(4)	3	6	9

GROUP Z

Blocks				Blocks			
(1)	5	9	11	(1)	5	9	11
(2)	3	7	12	(2)	3	7	12
(3)	1	6	10	(3)	1	6	10
(4)	2	4	8	(4)	2	4	8

Table II serves a double purpose. The body of the table gives the pattern of the arrangement in the blocks of the $k(k - 1)$ varieties, the V's being simply symbols for varieties. However, if each V is regarded as the total of the two observations on a variety which runs in the two replicates of the group, then this table is one which is made up in the course of the analysis.

The use of the Latin squares for getting the arrangement in the third group is perfectly general except for squares which have the property of the 6 by 6 and 10 by 10 Latin square. The solution of the lattice is simplified if the columns are interchanged in the Latin square so as to put the A. B. C.—in order along the main diagonal.

The mathematics for these statistical designs is similar to that used in the development of the rectangular lattices. Following the method used in the author's publication, *Rectangular Lattices*, the necessary formulas and equations are evolved for the analysis of the triple rectangular lattices. The symbols used in the paper are defined as follows:

1. R_{hj} is the sum of the yields of the varieties for replicate j of group h.

TABLE II

GROUP X

Blocks				
(1)	0	V_{123}	V_{134}	V_{142}
(2)	V_{214}	0	V_{231}	V_{243}
(3)	V_{312}	V_{324}	0	V_{341}
(4)	V_{413}	V_{421}	V_{432}	0

GROUP Y

Blocks				
(1)	0	V_{214}	V_{312}	V_{413}
(2)	V_{123}	0	V_{324}	V_{421}
(3)	V_{134}	V_{231}	0	V_{432}
(4)	V_{142}	V_{243}	V_{341}	0

GROUP Z

Blocks				
(1)	0	V_{231}	V_{341}	V_{421}
(2)	V_{142}	0	V_{312}	V_{432}
(3)	V_{123}	V_{243}	0	V_{413}
(4)	V_{134}	V_{214}	V_{324}	0

TABLE III

A	C	D	B
D	B	A	C
B	D	C	A
C	A	B	D

2. A_{hi} is the difference between the yields of the varieties in blocks i of group h.
3. B_{hi} is the total of the yields of the varieties from the blocks i of group h.
4. T_{hi} is the sum of the yields from all six replicates of the varieties listed in block i of group h.
5. V_{efg} is the sum of the yields from all six replicates of the variety with subscript efg.
6. G is the grand total of the yields of all the varieties in the six replicates.
7. y_{efg} is the yield of a variety with subscript efg for a particular replicate and a particular group.
8. k is the number of blocks in a replicate.

TABLE IV

Analysis of Variance Table

Source of Variation	Degrees of Freedom	Sum of Squares	
Replicates	5	$\sum_{h}^{x,y,z} \sum_{j=1}^{2} \frac{R_{hj}^2}{k(k-1)} - \frac{G^2}{6k(k-1)}$	(3)
Component (a)	$3(k-1)$	$\sum_{h}^{x,y,z} \sum_{i=1}^{k} \frac{A_{hi}^2}{2(k-1)} - \frac{\sum_{h}^{x,y,z} (R_{h1} - R_{h2})^2}{2k(k-1)}$	(4)
Component (b)	$3(k-1)$	$\frac{1}{12k(2k-3)} \left\{ \sum_{h}^{x,y,z} \sum_{i=1}^{k} (2k-1)(3B_{hi} - T_{hi})^2 \right.$	(5)
		$- 2 \sum_{i=1}^{k} [(3B_{xi} - T_{xi})(3B_{yi} - T_{yi}) + (3B_{xi} - T_{xi})(3B_{zi} - T_{zi})$ $\left. + (3B_{yi} - T_{yi})(3B_{zi} - T_{zi})] - 2 \sum_{h}^{x,y,z} [3(R_{h1} + R_{h2}) - G]^2 \right\}$	(6)
Varieties (unadjusted)	$k(k-1) - 1$	$\frac{\sum V_{efg}^2}{6} - \frac{G^2}{6k(k-1)}$	(7)
Error (intra-block)	$5k^2 - 11k + 1$	by subtraction	
Total	$6k(k-1) - 1$	$Sy_{efg}^2 - \frac{G^2}{6k(k-1)}$	(8)

In the method of analysis used, the variety means are adjusted by using both the inter- and intra-block variance. This necessitates finding two weights W and W'. These weights are also used in calculating the standard errors and the efficiency of the design.

The weights are calculated by two simple formulas

$$\frac{1}{W} = E \tag{1}$$

and

$$\frac{1}{W'} = \frac{6B - E}{5} \tag{2}$$

where B is the average mean square for component (a) and (b),
E is the mean square for the error (intra-block).

The analysis of variance table for this design is given in Table IV.

The adjusting of the varietal means using both the inter- and the intra- block information is accomplished by calculating certain constants. These constants are then subtracted from the varietal means. If the varietal means are arranged in the order of group X and with the Latin letters superimposed, then the constants to be subtracted from row i, column i, and Latin letter i are as follows:

$$c_{xi} = \frac{(W-W')\{[k(2W+W')-3W'](3B_{xi}-T_{xi})-(W-W')\sum_{h}^{x,y,z}(3B_{hi}-T_{hi})\}}{6[k(2W+W')-3W][k(2W+W')-3W']} \tag{9}$$

$$c_{yi} = \frac{(W-W')\{[k(2W+W')-3W'](3B_{yi}-T_{yi})-(W-W')\sum_{h}^{x,y,z}(3B_{hi}-T_{hi})\}}{6[k(2W+W')-3W][k(2W+W')-3W']} \tag{10}$$

$$c_{zi} = \frac{(W-W')\{[k(2W+W')-3W'](3B_{zi}-T_{zi})-(W-W')\sum_{h}^{x,y,z}(3B_{hi}-T_{hi})\}}{6[k(2W+W')-3W][k(2W+W')-3W']}. \tag{11}$$

To test the significance of two varieties the variance of their difference is needed. In the triple rectangular lattice designs there are seven formulas for the variances depending upon the combination of the varietal means.

The formulas for the variances of two varieties appearing together in same block with

1 digit alike in the subscripts
as

Variance $(V_{125} - V_{134})$

$$= \frac{1}{3W}\left\{1 + \frac{2(W - W')[k(2W + W') - (W + 2W')]}{[k(2W + W') - 3W][k(2W + W') - 3W']}\right\}; \tag{12}$$

2 digits alike in the subscripts
as

Variance $(V_{125} - V_{421})$

$$= \frac{1}{3W}\left\{1 + \frac{(W - W')[2k(2W + W') - (W + 5W')]}{[k(2W + W') - 3W][k(2W + W') - 3W']}\right\}; \tag{13}$$

3 digits alike in the subscripts
as

$$\text{Variance } (V_{125} - V_{152}) = \frac{1}{3W}\left\{1 + \frac{2(W - W')}{k(2W + W') - 3W}\right\}. \tag{14}$$

The formulas for the variance of two varieties not appearing in same block

No digits alike in the subscripts
as

Variance $(V_{126} - V_{754})$

$$= \frac{1}{3W}\left\{1 + \frac{3(W - W')[k(2W + W') - (W + 2W')]}{[k(2W + W') - 3W][k(2W + W') - 3W']}\right\}; \tag{15}$$

1 digit alike in the subscripts
as

Variance $(V_{125} - V_{342})$

$$= \frac{1}{3W}\left\{1 + \frac{(W - W')[3k(2W + W') - (2W + 7W')]}{[k(2W + W') - 3W][k(2W + W') - 3W']}\right\}; \tag{16}$$

2 digits alike in the subscripts
as

Variance $(V_{125} - V_{231})$

$$= \frac{1}{3W}\left\{1 + \frac{(W - W')[3k(2W + W') - (W + 8W')]}{[k(2W + W') - 3W][k(2W + W') - 3W']}\right\}; \tag{17}$$

3 digits alike in the subscripts
as

$$\text{Variance } (V_{126} - V_{612}) = \frac{1}{3W}\left[1 + \frac{3(W - W')}{k(2W + W') - 3W}\right]. \tag{18}$$

To illustrate numerically the method of analysis for the triple rectangular lattices the analysis is presented of an alfalfa variety experiment[1] performed in 1948 at the Piedmont Field Station near Orange, Virginia.

There were 12 varieties to be tested in the experiment so a 4 × 3 rectangular lattice was used. The experiment was set up as in Table I and when placed in the field, the varieties were randomized within each block and the blocks within each replicate.

After the first cutting of the alfalfa, the yields were tabulated and compiled for computational purposes as shown in Table VI. The upper figure in each plot refers to the variety number while the lower figure is the variety yield in pounds per plot (green weight). The size of the plot was 2 × 20 feet.

In Table VII the yields of each variety have been totaled by groups, that is, yields of the same variety in replicate 1 and 2 of group X were added together, etc.

The total yields of the 12 varieties are given in Table VIII. Here the three group yields for each variety were added together and the rows, columns, and Latin letter totals recorded.

The calculations for the analysis of variance are derived by substituting in formulas (3) through (8).

In order to substitute in formulas (4) and (5) (the block components sums of squares) it is convenient to form Table IX.

The component (a) set of differences are the differences between the sums of yields for the 3 varieties appearing together in block i of replicate

[1]The alfalfa data are used through the courtesy of Agronomists T. J. Smith and G. D. Jones of the Virginia Agricultural Experiment Station. Mrs. Sally R. Hudgins of the Virginia Agricultural Experiment Station is responsible for the numerical analysis.

TABLE VI

YIELD OF ALFALFA VARIETIES BY REPLICATIONS

GROUP *X*

Blocks	Replicate I				Block Totals	Blocks	Replicate 2				Block Totals
(1)	0	1 13.06	2 5.68	3 6.28	25.02	(1)	0	1 10.70	2 4.36	3 5.66	20.72
(2)	4 8.24	0	5 8.32	6 7.84	24.40	(2)	4 10.34	0	5 6.44	6 10.06	26.84
(3)	7 7.32	8 6.86	0	9 5.04	19.22	(3)	7 5.62	8 7.90	0	9 7.70	21.22
(4)	10 8.88	11 11.42	12 10.38	0	30.68	(4)	10 6.46	11 8.36	12 6.74	0	21.56
Total				(R_{x1})	99.32					(R_{x2})	90.34

GROUP *Y*

Blocks	Replicate 1				Block Totals	Blocks	Replicate 2				Block Totals
(1)	0	4 8.55	7 9.72	10 4.00	22.27	(1)	0	4 10.74	7 8.18	10 8.92	27.84
(2)	1 10.56	0	8 6.60	11 9.64	26.80	(2)	1 12.62	0	8 8.52	11 11.92	33.06
(3)	2 6.76	5 8.60	0	12 8.06	23.42	(3)	2 9.18	5 9.76	0	12 8.34	27.28
(4)	3 7.60	6 7.82	9 7.98	0	23.40	(4)	3 7.76	6 10.38	9 10.70	0	28.84
Total				(R_{y1})	95.89					(R_{y2})	117.02

GROUP Z

Blocks	Replicate 1				Block Totals	Blocks	Replicate 2				Block Totals
(1)	0	5 9.86	9 9.28	11 13.04	32.18	(1)	0	5 8.68	9 9.40	11 9.98	28.06
(2)	3 8.74	0	7 9.34	12 10.68	28.76	(2)	3 5.46	0	7 9.41	12 10.52	25.39
(3)	1 11.36	6 8.52	0	10 6.32	26.20	(3)	1 14.02	6 11.76	0	10 8.84	34.62
(4)	2 5.54	4 10.58	8 8.88	0	25.00	(4)	2 8.96	4 12.00	8 9.64	0	30.60
Total				(R_{z1})	112.14					(R_{z2})	118.67

TABLE VII

VARIETY YIELDS BY GROUPS

GROUP X

				B_{xi}	$3(B_{xi})$
0	1 23.76	2 10.04	3 11.94	45.74 (B_{x1})	137.22
4 18.58	0	5 14.76	6 17.90	51.24 (B_{x2})	153.72
7 12.94	8 14.76	0	9 12.74	40.44 (B_{x3})	121.32
10 15.34	11 19.78	12 17.12	0	52.24 (B_{x4})	156.72
Group totals				189.66	568.98

TABLE VII—*Continued*

GROUP Y				B_{yi}	$3(B_{yi})$
0	4 19.29	7 17.90	10 12.92	50.11 (B_{y1})	150.33
1 23.18	0	8 15.12	11 21.56	59.86 (B_{y2})	179.58
2 15.94	5 18.36	0	12 16.40	50.70 (B_{y3})	152.10
3 15.36	6 18.20	9 18.68	0	52.24 (B_{y4})	156.72
Group totals				212.91	638.73

GROUP Z				B_{zi}	$3(B_{zi})$
0	5 18.54	9 18.68	11 23.02	60.24 (B_{z1})	180.72
3 14.20	0	7 18.75	12 21.20	54.15 (B_{z2})	162.45
1 25.38	6 20.28	0	10 15.16	60.82 (B_{z3})	182.46
2 14.50	4 22.58	8 18.52	0	55.60 (B_{z4})	166.80
Group totals				230.81	692.43

1 of a group h and the sums of the yields of the same varieties appearing in block i of replicate 2 of the same group. The component (b) set of differences is obtained by subtracting the T_{hi} values of Table VIII from the 3 B_{hi} values of Table VII. The two sets of differences are given in Table IX.

TABLE VIII
VARIETY TOTAL YIELDS

				Row Totals	Latin Letter Totals
0	1 72.32	2 40.48	3 41.50	154.30 (T_{x1})	A 166.12 (T_{z1})
4 60.45	0	5 51.66	6 56.38	168.49 (T_{x2})	B 145.81 (T_{z2})
7 49.59	8 48.40	0	9 50.10	148.09 (T_{x3})	C 172.12 (T_{z3})
10 43.42	11 64.36	12 54.72	0	162.50 (T_{x4})	D 149.33 (T_{z4})
Column Totals 153.46 (T_{y1})	185.08 (T_{y2})	146.86 (T_{y3})	147.98 (T_{y4})	633.38 (G)	633.38

The results of the analysis of variance for the experiment are shown in Table X.

In adjusting the variety means using the inter- and intra-block variance, the weights and correction terms are calculated from formulas (1) and (2) and formulas (9), (10), and (11), respectively. W was found to be .75632 and W' to be .35427. The unadjusted variety means and the correction terms (c_{xi}, c_{yi}, and c_{zi} values) are shown in Table XI. Subtracting the appropriate corrections from each unadjusted variety mean gives the adjusted variety means shown in Table XII.

The standard error of the difference between the means of two varieties occurring together in the same block and

(1) having one digit alike in the subscripts is .710;
(2) having two digits alike in the subscripts is .712;
(3) having three digits alike in the subscripts is .713.

The standard error for two varieties not occurring together in the same block and

(1) having one digit alike in the subscripts is .734;
(2) having two digits alike in the subscripts is .735.

TABLE IX

COMPONENT (a) BLOCK DIFFERENCES

Group X		Group Y		Group Z	
(A_{x1})	4.30	(A_{y1})	−5.57	(A_{z1})	4.12
(A_{x2})	−2.44	(A_{y2})	−6.26	(A_{z2})	3.37
(A_{x3})	−2.00	(A_{y3})	−3.86	(A_{z3})	−8.42
(A_{x4})	9.12	(A_{y4})	−5.44	(A_{z4})	−5.60
$(R_{x1} - R_{x2})$	8.98	$(R_{y1} - R_{y2})$	−21.13	$(R_{z1} - R_{z2})$	−6.53

COMPONENT (b) BLOCK DIFFERENCES

$3B_{xi} - T_{xi}$		$3B_{yi} - T_{yi}$		$3B_{zi} - T_{zi}$	
(x_1)	−17.08	(y_1)	−3.13	(z_1)	14.60
(x_2)	−14.77	(y_2)	−5.50	(z_2)	16.64
(x_3)	−26.77	(y_3)	5.24	(z_3)	10.34
(x_4)	−5.78	(y_4)	8.74	(z_4)	17.47
Totals	−64.40		5.35		59.05

TABLE X

THE ANALYSIS OF VARIANCE OF A TRIPLE RECTANGULAR LATTICE EXPERIMENT

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Replicates	5	59.216	11.843
Component (a)	9	35.745	3.972
Component (b)	9	10.562	1.174
Blocks (eliminating varieties)	18	46.307	2.5726(B)
Varieties (unadjusted)	11	165.899	15.082
Error	37	48.920	1.3222(E)
Total	71	320.342	

REFERENCES

[1] G. M. Cox, R. C. Eckhart and W. G. Cochran, "The Analysis of Lattice and Triple Lattice Experiments in Corn Varietal Tests", *Iowa Agri. Exp. Sta. Res. Bul.*, Vol. 281 (1940).

[2] F. Yates, "A New Method of Arranging Variety Trials Involving a Large Number of Varieties", *Journal Agri. Sci.*, Vol. 26 (1936) pp. 424–455.

TABLE XI
UNADJUSTED VARIETY MEAN YIELDS

					c_{xi}	c_{zi}
		1 (C)	2 (D)	3 (B)		
	0	12.053	6.745	6.916	(c_{x1}) − .216	(A) .193 (c_{z1})
	4 (D)		5 (A)	6 (C)		
	10.075	0	8.610	9.396	(c_{x2}) − .187	(B) .217 (c_{z2})
	7 (B)	8 (D)		9 (A)		
	8.265	8.066	0	8.350	(c_{x3}) − .336	(C) .134 (c_{z3})
	10 (C)	11 (A)	12 (B)			
	7.236	10.726	9.120	0	(c_{x4}) − .091	(D) .209 (c_{z4})
c_{yi}	(c_{y1}) −.036	(c_{y2}) −.068	(c_{y3}) .077	(c_{y4}) .096		

TABLE XII
ADJUSTED VARIETY MEAN YIELDS

Variety	Mean	Variety	Mean
1.	12.20	7.	8.42
2.	6.68	8.	8.26
3.	6.82	9.	8.40
4.	10.09	10.	7.23
5.	8.53	11.	10.69
6.	9.35	12.	8.92

REFERENCES CONTINUED

[3] F. Yates, "The Recovery of Inter-Block Information in Three Dimensional Lattice", *Annals of Eugenics*, Vol. 9 (1939) pp. 136–156.

[4] F. Yates, "The Recovery of Inter-Block Information in Balanced Incomplete Block Designs", *Annals of Eugenics*, Vol. 10 (1940) pp. 317–325.

[5] Harshbarger, Boyd, "Rectangular Lattices", *Virginia Agricultural Experiment Station, Memoir 1*, 1947.

PROBLEMS OF THE OPTIMUM CATCH IN SMALL WHITEFISH LAKES

Richard B. Miller

The whitefish fishery of the Canadian prairies is pursued in hundreds of widely scattered lakes, most of which are small and shallow, but highly productive. The whitefish, *Coregonus clupeaformis* Mitchill, thrives in them, despite the fact that they warm to the bottom every summer and produce heavy algal blooms. In the province of Alberta, these lakes range in size from ten to over four hundred square miles in area and each produces from fifty thousand to over half a million pounds of whitefish per year. The annual catch of whitefish from these lakes is governed by the quota system; each lake has a limit set for each fishing season; when the limit has been taken, the local fish inspector closes the lake. If, at the time the lake is closed, the fishing is still good and the season not advanced, it is common practice to grant extensions and re-open lakes for a further limit.

From time to time, one or another of these lakes fails to produce whitefish of commercial size when the fishing season opens. The lake is declared to be "fished out" and is closed for one or more years until tests show it to be yielding good fish again. Heavy plantings of "eyed" eggs or fry are made to help the restoration.

These trial and error tactics have worked fairly well, and, over a period of twenty-five years, administrators have arrived at a fair estimate of the sustained annual yield of each lake. However, this type of administration does not provide the answers to several important questions. First, it is not possible to predict when a particular fishery may collapse, since it cannot be shown that any particular collapse is due to overfishing. Second, the optimum catches—the reaching of a proper balance between recruitment and mortality—are unknown and probably differ from the actual catches. Third, the effect, if any, of the plantings of "eyed" eggs or fry is not known.

It was partly from a desire to answer these questions that the investigation reported in this paper was begun; but, more important, these small lakes seemed to offer splendid opportunities to observe the effects of fishing pressure on populations which were known to be homogeneous. In these small lakes, the picture is not complicated by the possibility of immigration or emigration; the whitefish being caught in any lake are from a population that arose in that lake, is the sole whitefish population of the lake, and cannot be augmented by arrivals from outside populations or depleted by other than fishing or natural mortality. Thus many of the variables which complicate the study of a marine fishery, or fisheries of large lakes, are conveniently absent.

For the past seven years, the author, in co-operation with the provincial fisheries administration, has been taking samples of the commercially caught whitefish from a series of lakes and analyzing them for age composition and rate of growth. We hoped that we should be able to recognize overfishing or underfishing and assess the value of the whitefish hatchery. In this paper, I wish to discuss the whitefish populations of two lakes; Lake Wabamun and Pigeon Lake. Lake Wabamun has been closed for part of the period of study, whereas Pigeon Lake has been deliberately over-exploited. The whitefish populations of the two lakes thus form an interesting contrast, and clearly show the effects of fishing pressure. The lakes are similar in size (35–40 square miles) and have a maximum depth of 35 feet. The common sucker *(Catostomus commersonnii)* is the only whitefish competitor in both lakes. Both lakes are fished with gill nets of $5\frac{1}{2}$ inch (stretched measure) mesh. Samples of the commercial catch have been taken twice each year; approximately three thousand fish have been measured and their ages determined. Calculations of the lengths of the fish at the end of each year of life were made by Van Oosten's method (1923).

THE LAKE WABAMUN FISHERY

The catches of whitefish from Lake Wabamun were from 150,000 to 200,000 pounds annually from 1918 to 1935 (Table 1); the years 1935 to 1940 saw greatly increased yields—up to 600,000 pounds, some three times as great as the average for the previous eighteen years. In 1941 the fishery collapsed; in spite of high effort, a low yield of small fish resulted. (see Table 1). As a consequence, the lake was closed for two years. In 1944 it was reopened and the catch has risen to a high

TABLE 1

TOTAL CATCH, TOTAL NETS, NUMBER OF MEN ENGAGED AND CATCH PER NET-MAN IN LAKE WABAMUN

Year	Catch (lbs.)	No. Nets	No. men	Catch per net-man
1918	146,000	180	46	17.3
1919	134,500	96	35	40.0
1920	no data	—	—	—
1921	188,500	110	25	68.4
1922	141,000	202	46	15.2
1923	187,400	306	54	11.3
1924	309,500	202	63	24.3
1925	273,200	341	98	8.2
1926	172,900	411	107	3.9
1927	55,400	366	85	1.8
1928	74,300	365	66	3.1
1929	213,700	282	72	10.5
1930	155,700	384	64	6.3
1931	206,000	408	67	7.5
1932	189,300	486	81	4.8
1933	194,900	510	85	4.5
1934	no data	—	—	—
1935	241,000	846	207	1.4
1936	387,000	808	118	4.1
1937	653,200	983	983	0.7
1938	599,700	1439	1439	0.3
1939	521,600	3681	862	0.2
1940	334,200	1186	643	0.4
1941	174,300	764	382	0.6
1942	closed	—	—	—
1943	closed	—	—	—
1944	106,000	422	422	0.6
1945	318,900	791	791	0.5
1946	349,820	997	997	0.4
1947	532,780	1299	1299	0.3

level again. In the absence of age and growth data, it is impossible to state whether the collapse in 1941 was or was not due to overfishing.

Samples of the fishery were first taken in 1942 and have been taken regularly since. The age composition of these samples is shown in Table 2. In the year following the collapse of the fishery, the sample contained 69 percent of four-year-old fish; during the next four years, the samples contained more and more older fish—by the fall of 1947, only one percent of the sample was of four-year or younger fish.

TABLE 2

AGE COMPOSITION OF SAMPLES OF WHITEFISH FROM LAKE WABAMUN

Date	Size of sample	Percent of each age							
		2	3	4	5	6	7	8	9
January '42	34	0	0	69	23	8	0	0	0
Aug. '42–Feb. '43	73	0	5	43	43	7	2	0	0
June '43–Mar. '44	633	0	3	48	39	10	0	0	0
July '44–Jan. '45	251	0	0	14	54	28	4	0	0
July–Aug. '45	201	0	0	0.5	16	58	23	2	0.5
October '46	100	0	1	0	4	66	23	4	2
February '47	100	0	7	22	32	25	9	5	0
Sept.–Oct. '47	371	0	0	1	3	19	49	22	5
	1763								

In Table 3 the rates of growth of the year classes in the samples are shown; year classes from 1935 to 1943 are represented. During the period of closure and light fishing, there has been no change in growth rate. The apparent increase in growth of the later year classes (1942–1943) is mainly due to selective action of the gill nets.

Lake Wabamun shows us, then, that a fishery which is recovering from heavy exploitation has, first, a steady growth rate, and second, an increasing number of older fish in the population.

TABLE 3

CALCULATED TOTAL LENGTH (MM.) OF EACH YEAR CLASS AT THE END OF EACH YEAR OF LIFE IN LAKE WABAMUN

Year Class	No. Fish	1	2	3	4	5	6	7	8
1935	2	100	203	305	364	400	421	438	—
1936	4	104	205	292	346	383	406	431	—
1937	33	106	209	288	344	380	410	432	449
1938	134	109	211	291	348	378	409	429	—
1939	134	112	210	294	347	383	406	428	461
1940	102	109	207	288	345	376	400	423	—
1941	21	112	202	269	333	370	405	—	—
1942	4	123	227	321	364	379	—	—	—
1943	5	112	222	311	—	—	—	—	—

THE PIGEON LAKE FISHERY

In the absence of growth and age composition data prior to the collapse of the Lake Wabamun fishery in 1941, it is impossible to say whether or not it was due to overfishing. It was decided to allow increased fishing in Pigeon Lake and follow the changes in the population which preceded a collapse of the fishery. Pigeon Lake has had a fairly steady yield of 150,000 to 200,000 pounds per year since 1918 (Table 4). In 1941 the catch was increased to nearly 600,000 pounds,

TABLE 4

TOTAL CATCH, TOTAL NETS, NUMBER OF MEN ENGAGED AND CATCH PER NET-MAN IN PIGEON LAKE

Year	Catch (lbs.)	No. nets	No. men	Catch per net-man
1918	78,700	195	49	8.2
1919	144,000	64	60	37.5
1920	no data	—	—	—
1921	138,000	330	60	7.0
1922	152,000	290	75	7.0
1923	183,720	316	87	6.7
1924	228,700	700	102	3.2
1925	277,600	700	129	3.1
1926	248,200	995	162	1.5
1927	235,700	858	142	1.9
1928	144,800	525	167	1.6
1929	146,000	570	95	2.7
1930	148,000	684	114	1.9
1931	130,900	810	135	1.2
1932	196,300	810	135	1.8
1933	214,900	1128	188	1.0
1934	no data	—	—	—
1935	182,800	374	374	1.3
1936	135,300	284	284	1.7
1937	115,800	169	169	4.0
1938	179,000	451	262	1.5
1939	203,100	1405	265	0.5
1940	269,500	1405	228	0.8
1941	582,900	878	421	1.6
1942	354,600	633	293	1.9
1943	340,000	534	487	1.3
1944	485,000	354	260	5.3
1945	411,000	1095	826	0.5
1946	350,000	1232	1032	0.3
1947	160,000	798	798	0.3

over three times the previous annual average. This increased yield continued for six years; the fishery then collapsed in 1947 and the lake is now closed. It is interesting, and perhaps significant, that Lake Wabamun also collapsed after the same period, six years, of heavy fishing.

In Table 5 the age compositions of samples of the catch since 1942 are shown. In 1942 the average age was 5.1 years and ninety percent of the sample was of four-year-olds or older. With the increased fishing, the samples contained more and more younger fish; by the fall of 1947 nearly three-quarters of the catch was of two-year-olds and the fishery had collapsed.

TABLE 5

AGE COMPOSITION OF SAMPLES OF WHITEFISH FROM PIGEON LAKE

Date	No. fish	Percent of each age								
		1	2	3	4	5	6	7	8	9
Aug. '42	100	0	8	2	17	28	37	8	0	0
Dec. '43–Feb. '44	330	0	0	27.3	25.5	30.3	13.2	3.7	0	0
Aug. '44	127	.8	0	1.6	33	26	26.8	11	.8	0
Sept. '45	102	1	0	11.8	52.9	23.5	8.8	·2	0	0
Dec. '46–Jan. '47	301	0	13.3	21.6	40.2	13.6	7.6	3.7	0	0
Oct. '47	215	0	74.5	20.9	3.3	0.4	0.4	0.4	0	0
	1175									

In Table 6 the rates of growth of each year class from 1935 to 1945 are shown. It is clear that with the increased fishing pressure, the growth rate has also increased. For example, the calculated length of a three-year-old increased from 285 mm. for the year class of 1935 to 333 mm. for the 1942 year class. The picture is more easily seen by examining Table 7. This table shows the weight of 100 whitefish for each year of the fishery since 1942; the weights were calculated by multiplying the average weight of each age group by the percent abundance of that age group in the sample. Table 7 also shows the average increments in total length for the whole population beyond the 1942 level, and the percent of the catch made up of fish four years old and younger. Maximum efficiency—greatest weight per hundred fish—was reached in 1944, the third year of the increased fishing, while maximum growth was not reached until two years later. The fast

TABLE 6

CALCULATED TOTAL LENGTH (MM.) OF EACH YEAR CLASS AT THE END OF EACH YEAR OF LIFE IN PIGEON LAKE

Year Class	No. Fish	1	2	3	4	5	6	7	8
1935	9	102	203	285	332	371	396	415	—
1936	21	101	196	280	330	367	391	432	—
1937	23	106	215	306	349	381	421	447	—
1938	29	103	208	295	353	391	419	444	473
1939	69	108	210	308	364	403	444	459	—
1940	83	111	213	310	372	417	431	439	—
1941	32	114	222	318	383	403	—	—	—
1942	27	120	229	333	375	389	—	—	—
1943	32	118	232	330	363	—	—	—	—
1944	39	119	255	314	—	—	—	—	—
1945	24	122	252	—	—	—	—	—	—

TABLE 7

AGE COMPOSITION OF CATCH, WEIGHT (OUNCES) OF 100 FISH FROM THE CATCH AND AVERAGE ANNUAL GROWTH INCREMENT (MM. TOTAL LENGTH); PIGEON LAKE, 1942-1947

Year	Percent of catch 4 years and under	Calculated weight of 100 fish	Average annual growth increment
1942	27	3014	0
1943	52.8	2881	13.2
1944	35.4	3850.3	22
1945	65.8	3555	41
1946	75.1	2860.4	49.2
1947	98.7	1035.2	17.8

growth of 1946 failed to compensate for the youth of the catch, which was of 75 percent four-year-fish and younger.

We may conclude from the Pigeon Lake study that:

(1) The fishery collapsed due to overfishing.

(2) The overfishing resulted in increased growth rate.

(3) At first, the increased growth rate more than compensated for younger fish in the catch, i.e. the average weight of the fish increased. After three years, the increasing number of young fish "caught up" to the increasing growth rate, i.e. although the growth rate increased for

another two years, the efficiency began to decrease as the fish were taken too young. This decrease in efficiency took place when more than 35 percent of the catch consisted of fish four years old and younger.

(4) In the final year—the year of collapse—the rate of growth decreased.

CATCH PER UNIT EFFORT

In these small whitefish lakes, the total catch is determined arbitrarily; also there is no control over the number of licenses sold. As a consequence, catch per unit of effort fluctuates inversely with number of licenses and the price of whitefish.

Tables 1 and 4 show the total catches, total men fishing, total nets and catch per net-man in Lakes Wabamun and Pigeon since 1918. In general, it may be seen that catch per net-man is low when the catch is high and vice versa. During the last three years in Lake Wabamun, catch per net-man has been very low; yet from the age composition data we know that the fishery is in good order and that there is no immediate danger of collapse. In Pigeon Lake, the lowest catches per net-man (0.5 and 0.8 in 1939 and 1940) preceded the largest catch in the lake's history, nearly six hundred thousand pounds in 1941. Much more precise statistics than numbers of men and numbers of nets are needed to give catch-effort data of any value. Even if better statistics were available, it is doubtful if they would help much in these fisheries; it is a common observation among fishermen that these lakes yield heavily per net right up to, and including, the season before a collapse. It would appear that catch per net is more a measure of availability than of abundance.

THE EFFECT OF THE HATCHERY

The question arises, may the collapse of a fishery be avoided by the planting of "eyed" eggs or fry? The study of Pigeon Lake makes this appear unlikely. For it has been noted that in 1942, most of the fish spawned in Pigeon Lake as four-year-olds—at the end of their fifth summer. In 1946, all the two-year-olds were spawners; with the increased growth rate, younger and younger fish became mature. The collapse of the fishery was not due, therefore, to the failure of the fish to reproduce. There are plenty of one- and two-year-old fish in the lake now. These observations are perhaps enough to answer the question but we have tried to prove the answer by comparing the strengths of year classes which have had hatchery support with those which have

not. A more complete account of this work has been published elsewhere (Miller, 1946).

Table 8 shows a comparison of the year classes of 1940 and 1941 in four lakes. The figures in the table show the percentage of fish of each year class found in the samples during six years of sampling. Note that the year class of 1940 is stronger in each lake whether or not it received support from the hatchery. In Tables 9 and 10, the strengths of various other year classes are compared; again note that year class strength appears independent of hatchery operations.

There is no evidence, then, that the hatchery can influence production.

TABLE 8

THE RELATIVE STRENGTHS OF THE WHITEFISH YEAR CLASSES OF 1940 AND 1941 IN FOUR ALBERTA LAKES

Lake	Year Class	Hatchery plant	Percent of catch at each age						Total
			2	3	4	5	6	7	
Pigeon	1940	5 million	8	27.3	33	23.5	7.6	0.4	99.8
	1941	none	0	1.57	52.9	13.6	0.4	—	68.5
Buck	1940	none	7.7	45	33.7	40	—	—	126.4
	1941	1 million	0	3.5	10	—	8.4	—	21.9
Lesser Slave	1940	41 million	0	11.9	41.7	37.2	—	—	90.8
	1941	30 million	0	10.5	20.8	—	4.0	—	35.3
Wabamun	1940	5 million	0	3.35	11.7	15.4	45.5	—	76.0
	1941	6 million	0	0	0.5	18	18.6	—	37.1

TABLE 9

THE RELATIVE STRENGTHS OF FOUR WHITEFISH YEAR CLASSES IN PIGEON LAKE

Year Class	Hatchery plant	Percent of catch at each age			Total
		4	5	6	
1938	4 million	17	30.3	26.8	74.1
1939	3 million	25.5	26.0	8.8	60.3
1940	5 million	33.0	23.5	7.6	64.1
1941	none	52.9	13.6	0.4	66.9

TABLE 10
THE RELATIVE STRENGTHS OF TWO YEAR CLASSES OF WHITEFISH IN LESSER SLAVE LAKE

Year Class	Hatchery plant	Percent of catch at each age						Total
		5	6	7	8	9	10	
1936	none	27.2	50	5.2	7.8	0.4	0	90.6
1937	88.7 million	33.4	17.9	7.0	4.4	—	0	62.7

DISCUSSION

The data derived from the study of the fisheries of Lake Wabamun and Pigeon Lake make a number of conclusions possible, most of which are already known from research on marine fisheries. (For information on marine fisheries, I have leaned heavily on E. S. Russell's excellent little book, The Overfishing Problem (1942)).

(1) In a fishery in which heavy exploitation has ceased, the average age of the population increases and the growth rate is slow and steady; (Lake Wabamun).

(2) In a heavily exploited fishery, the average age of the population decreases and the growth rate increases; (facts clearly demonstrated over twenty-five years ago by Petersen (1922) for the plaice fisheries of the Kattigat, Belts and Baltic). We have seen that this increased growth rate at first more than compensates for the decreasing average age, i.e. the average weight of fish in the catch increases; but as the number of young fish in the catch increases, the increased growth rate fails to compensate and the average weight of fish in the catch begins to fall. This may occur before catches begin to fall off, so we may conclude:

(3) Overfishing is not just decreased yield in the face of increasing effort; *it begins when decreasing average age overtakes increasing growth rate and before decreased yields are evident.* Another important conclusion is a corollary to this one:

(4) The fishery will not maintain production at maximum growth rate; in order to produce sufficient thinning of the population to permit maximum growth rate, fish of less than commercial size had to be taken, and the fishery collapsed. Curiously enough:

(5) At the time of collapse, the growth rate decreased. This may

be due to one- and two-year-olds surviving in enormous numbers when the older age groups were removed, thus producing a crowding effect.

(6) The collapse of the fishery was not due to brood failures and hatchery plants can do nothing to prevent such collapses or assist in recovery following a collapse. (In an earlier paper on these same fisheries, (Miller, 1947), I was much impressed by the ability of the whitefish to mature at younger and younger ages as the older fish were removed. It was evident that fishing with selective gear for large fish could not deplete the brood stock. But I was incorrect in suggesting that, in such a fishery, overfishing was not likely to occur).

Of recent years, there have been attempts to devise a formula or formulae by the use of which one may determine when to harvest a year class in order to get its maximum bulk; i.e. the optimum catch. Such a formula must contain expressions for natural mortality rate and fishing mortality rate, which diminish the population, and also for rate of growth and recruitment which add to the population bulk. Ricker has done much interesting and valuable work along these lines which he has brought together in a recent important paper (Ricker, 1948). The effects of overfishing in Pigeon Lake, which the present paper describes, serve to emphasize how difficult it is to evolve a satisfactory formula. In such a formula it is usually necessary to assume that the rates of growth, of recruitment and of natural mortality are reasonably stable. For example, Ricker (1945) has calculated the best minimum size for bluegills in Muskellunge Lake, Indiana; he has shown how this size will change with different degrees of fishing (p) but has retained the same value for natural mortality rate in most of his calculations. Using Jackson's method (1939) of determining the average ratio of the number of individuals of each age to the number one year younger, I have calculated the annual mortality in Pigeon Lake to vary over a range of fifty percent from 1942 to 1946. Gill net returns are admittedly unreliable for such a calculation, but the large variation does suggest that the rate of natural mortality is not stable but varies perhaps with fishing pressure.

Rate of recruitment, too, may well be a function of fishing pressure. Fish of recent age groups in Pigeon Lake appear to be so numerous that their growth rates are decreasing. This abundance was probably

caused by a greater survival beyond the fry stage due to the removal, by fishing, of the older age groups.

That rate of growth is not constant, but varies with population density, is well known and formulae for the optimum catch try to take account of this fact. This is not easy to do, however. In Pigeon Lake, I have calculated that the increasing rate of growth during the six years of overfishing caused each age group to be 19.3 percent heavier each year. This same rate of fishing caused the age composition to change in such a way that 35 percent per year was added to the group four years old and less in age. The yield to the fishermen in weight of fish was the resultant of these two rates; the increasing rate of growth increased the yield for two years and then the increasing youth of the catch (which itself caused the increasing growth rate) began to decrease the yield. The resultant of the two rates, therefore, was a collapse of the fishery after six years of overfishing.

It would appear, then, that all the factors which must be included in a calculation of the optimum catch are quite likely subject to variation with varied fishing pressure. Furthermore, in the lakes under discussion, the whitefish has no serious competitor, which, in the event of the depletion of the whitefish, might occupy its place in the economy of the lake. This would not be the case in most fisheries; the threat of the ascendancy of undesirable species is a real one in many fisheries, (e.g. The Great Lakes). And so we have still another unpredictable effect of fishing pressure which will complicate calculations of the optimum catch.

A crude calculation of the extent of overfishing and an estimate of the optimum catch in Pigeon Lake may be made as follows:

The average annual catch during the six years of overfishing was 421,000 pounds. Since it took six years for this rate of exploitation to produce a collapse, we may conclude that each year one-sixth too much was taken. Therefore, the proper (optimum?) annual catch should have been six-sevenths of 421,000 pounds or 361,000 pounds. If this calculation is correct, the annual average for the period 1918–1940 of 179,000 pounds was only half the possible sustained yield.

Of course, it is a rather drastic procedure to over-exploit a fishery until it collapses in order to discover what the annual catch should have been. It has taught us, however, that the age composition of the whitefish catch must not be forced below 35 percent four-year-old and younger fish, and that age compositions of this order give the greatest average weight of fish.

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REFERENCES

Jackson, C. H. N. 1939. The analysis of an animal population. *Journal of Animal Ecology 8:* 238–246.

Miller, R. B. 1946. Effectiveness of a whitefish hatchery. *Journal of Wildlife Management 10(4):* 316–322.

Miller, R. B. 1947. The effects of different intensities of fishing on the whitefish populations of two Alberta lakes. *Journal of Wildlife Management 11(4):* 289–301.

Petersen, C. G. J. 1922. On the stock of plaice and the plaice fisheries in different waters. *Rep. Danish Biol. Station XXIX.*

Ricker, W. E. 1945. A method of estimating minimum size limits for obtaining maximum yields. *Copeia,* June 30, 1945 (2): 84–94.

Ricker, W. E. 1948. Methods of estimating vital statistics of fish populations. *Indiana University Publications. Science Series 15,* 1948: 101 pp.

Russell, E. S. 1942. The overfishing problem. *Cambridge University Press.* 130 pp.

Van Oosten, J. 1923. The whitefishes (*Coregonus clupeaformis*). A study of the scales of whitefish of known ages. *Zoologica, Scientific Contributions N. Y. Zoological Society II(17):* 380–412.

STATISTICS OF A LAKE TROUT FISHERY

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SINCE 1936 THE lake trout fishery of lake Opeongo, Algonquin Park, Ontario, has been followed by means of a creel census. This census was undertaken as an experiment in methodology as well as to give information on the lake trout *Cristivomer namaycush* (Walbaum). Thus one of the main objects has been to determine to what extent a record of the fish caught, and of the circumstances surrounding their capture can yield a picture of the nature of the population under exploitation and of the changes that may take place in it. From time to time sections of the data collected have been analyzed, chiefly those relating to size and rate of capture (Fry and Kennedy 1937, Fry 1939[b], Fry and Chapman 1948). In this report the growth rate, age composition and fecundity are related to the statistics of catch with the express aim of building up from these data some logical picture of the lake trout population from which this fishery is drawn. Thus the object is to determine within whatever limits it may be possible to attain, the size of the population, its annual increment in weight, its spawning strength and the degree to which the fishery exploits it.

The procedure by which the Algonquin Park Creel Census is conducted and the forms used are described elsewhere (Fry 1939[b]). Opeongo lake is the main census station in Algonquin Park and it has been possible to obtain an almost complete record of the fishing in this lake since virtually all the people visiting it, except for transient canoe parties, reach it by a single road and come in contact with the census taker.

Many of the fish taken are brought to the landing at Sproule bay either by guests of the fishing lodge located there or by parties who make a special trip for a day's fishing. Most of the fish brought to the landing are available for measurement and examination. The census worker guts these fish in return for the privilege of examining them and in this way material is accumulated for stomach analysis and fecundity studies as well as lengths, weights and scale samples.

Lake Opeongo is situated in the Precambrian shield at 45° 42′ N. 78° 23′ W. It has an area of approximately 20 square miles and a maximum depth of about 175 feet. In one respect Opeongo has not been an ideal lake for this study. It is a unit in name only since it consists of four basins isolated from each other by shallow and constricted channels so that each basin has its own limnological peculiarities and perhaps its own discrete population of lake trout. In any event it is certain that lake trout are present in all the basins in summer and it is highly unlikely that they mix during that season. In consequence, therefore, all sections of the population are not affected equally in all years by the fishing that goes on. Fashions change, good luck in one of the arms at one time will swing the majority of the effort to that basin, perhaps for the rest of the season, and may leave the populations of the other basins relatively untouched for the time being. This was especially true during the war years when both gasoline and leisure were restricted and the anglers did little exploratory fishing. Because of the distinctness of these basins the Opeongo fishery, limnologically speaking, should be considered the yield of four small lakes rather than that of a single lake of moderate size. However, although these considerations may greatly affect specific conclusions regarding action to be taken in respect to the Opeongo fishery, they appear to have no great bearing on the general conclusions presented here.

ACKNOWLEDGMENTS

The Algonquin Park Creel Census was initiated in 1936 at the suggestion of Dr. W. J. K. Harkness at the time Director of the Ontario Fisheries Research Laboratory of the Department of Zoology, University of Toronto. The expense of the work was borne by this laboratory in the first year aided by the Department of Lands and Forests through the kind support of Mr. F. A. MacDougall. Subsequently a grant in aid of the census was received from the National Research Council of Canada through the National Committee of Fish Culture. In 1946 the work was transferred to the Research Division of the Department of Lands and

TABLE 1

CHARACTERISTICS OF THE LAKE TROUT CATCH REMOVED FROM LAKE OPEONGO IN THE YEARS 1936-1947 INCLUSIVE. THESE ESTIMATES ARE BASED ON THE ASSUMPTION THAT 80% OF THE TOTAL CATCH WAS RECORDED BY THE CREEL CENSUS.

Year T	Units effort (100 hrs.) $E(T)$	Estimated number of fish caught	Removal pounds	Pounds per acre	Average Age fish landed years	Availability number captured per 100 boat hours $C(T)$
1936	20.3	2600	9400	0.7	—	128
1937	22.4	2700	7450	0.56	7.98	121
1938	16.3	1650	3940	0.29	7.11	101
1939	13.8	1550	4350	0.32	7.65	112
1940	11.7	1400	3320	0.25	7.11	120
1941	11.3	1100	2520	0.19	6.94	97
1942	5.7	630	1550	0.12	7.12	111
1943	7.1	900	2330	0.17	7.74	126
1944	9.2	1050	3390	0.25	8.13	126
1945	14.0	1420	5600	0.42	9.03	101
1946	17.4	1220	3620	0.27	9.10	70
1947	12.3	885	2500	0.19	7.81	72

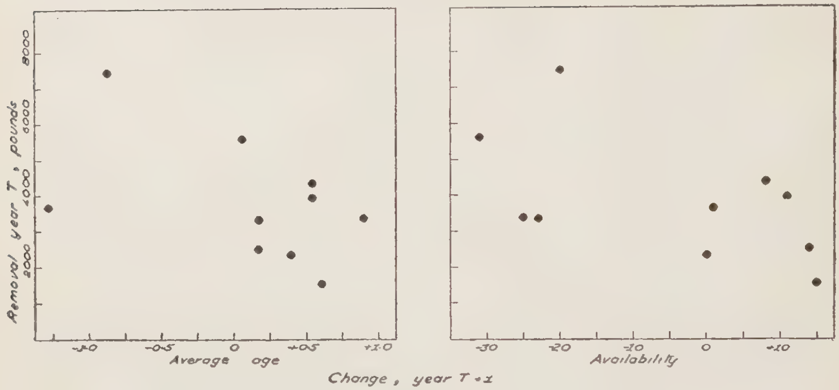
Forests of Ontario, which agency is now continuing it. The field work in the various years has been the responsibility of junior members of the staff of the Ontario Fisheries Research Laboratory and latterly of the Research Division of the Department of Lands and Forests. Among these were the following: Messrs. W. A. Kennedy, J. R. Brett, H. M. M. Tovell, D. W. Kirk, J. Long, R. R. Tasker, N. S. Baldwin and N. V. Martin. The writer is also indebted to Dr. D. B. DeLury and to Mr. D. Teichroew for much guidance and assistance in the mathematical analysis of these data. Mr. Teichroew performed most of the computing, as well as acting as mathematical consultant during the preparation of the paper. I am also indebted to Dr. J. M. Speirs for a critical reading of the manuscript.

STATISTICS OF REMOVAL

Introduction

The general statistics of removal of lake trout from lake Opeongo are given in table 1. During the period covered by the creel census the estimated number of lake trout captured per year has varied from 2700

FIGURE 1.



THE RELATION BETWEEN REMOVAL IN YEAR T AND THE CHANGE IN CHARACTERISTICS OF THE CATCH IN YEAR $T + 1$. THE COEFFICIENTS OF CORRELATION ARE: WITH CHANGE IN AGE $r = 0.554$; WITH CHANGE IN AVAILABILITY $r = 0.560$. THESE ARE ON THE BORDERLINE OF THE 0.05 LEVEL OF PROBABILITY. THE SCATTER IS PRESUMABLY DUE TO VARIATION IN THE STRENGTH OF THE INCOMING YEAR CLASSES.

in 1936 and 1937 to 630 in 1942. This variation has been the result of changes both in the size of the population and in the intensity of fishing effort. Previous to 1936 lake Opeongo was a relatively inaccessible place. It could be reached by car only by travelling a road which was the converted right of way of an old logging railway and this approach was from the more sparsely settled eastern section of the province. In 1936 a highway to the west was opened with the result that many more anglers from southern Ontario and the United States visited the lake, thus greatly increasing the rate at which the Opeongo trout population was exploited.

In each of the years 1936 and 1937 approximately 2000 boat hours of fishing effort were expended in the pursuit of lake trout in Opeongo. Lake trout in this district are typically taken by a troll on a metal line; in general two lines are fished from a boat at the same time. Fishing intensity waned somewhat in 1938 and dropped off still further with the beginning of the war. In 1942 the fishing effort expended was only about 600 boat hours, the least recorded. After the war the expenditure of fishing effort returned to approximately the 1938–39 level.

The estimated poundage of lake trout removed has varied from 1550 to 9400 pounds per year. These values represent an annual removal of from 0.12 to 0.7 pounds per acre of water surface. Although the removals are of a low order of magnitude in terms of yield per acre, they have had an influence on the fishery as figure 1 indicates. This graph shows the

correlation between removal in one year and two indices of the fishery in the following year.

Provisionally at least, these two correlations may be taken to indicate that fishing mortality is not negligible in comparison with natural mortality.

Age Composition of the Catch

A detailed estimate of the removal by age groups is given in table 2.

TABLE 2

ESTIMATED NUMBERS, $K(x;T)$, OF LAKE TROUT REMOVED AT VARIOUS AGES FROM LAKE OPEONGO DURING THE YEARS 1936 TO 1947. SEE TABLE 1 FOR TOTALS

Year of Capture	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
1936	30	95	128	233	474	665	478	260	118	57	19	25	10	15	11
1937	0	4	34	198	650	1025	555	176	38	4	8	8	0	0	0
1938	12	74	127	275	420	439	195	90	3	9	3	0	3	0	0
1939	39	36	116	221	321	393	223	90	47	24	13	15	4	4	4
1940	20	84	82	224	434	364	120	46	14	6	0	6	0	0	0
1941	8	79	144	275	235	200	104	22	11	11	11	0	0	0	0
1942	7	18	46	117	217	121	53	28	8	9	2	2	0	0	0
1943	6	42	42	121	272	211	133	42	0	24	6	0	0	0	0
1944	8	26	84	114	197	202	198	93	44	31	9	22	9	4	4
1945	0	11	32	69	170	352	373	159	84	37	21	43	16	37	16
1946	19	30	78	116	240	325	217	93	47	26	11	7	0	7	4
1947	3	30	55	85	217	221	153	76	18	12	3	0	6	3	3
Mean	11	40	76	165	306	350	212	83	29	18	8	9	3	5	3
Percentage	0.8	3.0	5.8	12.5	23.2	26.6	16.1	6.3	2.2	1.3	0.6	0.7	0.3	0.4	0.2

These estimates were made by determining the age composition of a sample of scales gathered as the opportunity presented itself throughout the fishing season. These scale samples represented 30% of the estimated catches. It is presumed that there was no bias in the collection of these samples although it is certain that this assumption is not strictly true, for as can be well imagined larger than average fish were more likely to be brought to our attention than smaller ones. However, since so large a sample was examined the effect of this bias cannot be very great. In the case of the estimate for 1936 this procedure was not followed since some of the scale samples were lost. The age composition of that year's catch was estimated from the age composition of each size class as estimated from the average growth in length over the eleven year period.

The Opeongo lake trout first enter the fishery at age III but only a negligible number of this age group are taken. Age groups V to X have provided 90% of the total fishery, ages VII and VIII making the most important contribution within this range. These two age groups have been responsible for 50% of the total fishery. The highest age that has been read is XVII. Fish of ages XV to XVII were taken only infrequently after 1936 except in 1945 when a markedly high number of old fish were taken. These no doubt were from the less accessible basins of the lake which were not so highly exploited during the war years.

The Virtual Population

When the total catch and its age composition are known for a number of years it is possible to sum up the contribution of each year class which has passed through the fishery in that time. This summation of the year class has to remain in abeyance until the year class has passed completely through the fishery so that no more of its members will be captured to add to the total. It is proposed that this complete contribution of a year class to the fishery shall be termed the *virtual population.* It is something which can be seen, piecemeal of course, but which is not the true population. However it is all that we are able to see of most populations of fish and it places one limit on a number of estimates. Moreover the value of these estimates is the greater the more intensive the fishery and hence the greater the need for giving it close statistical attention.

Estimates of the virtual population for various year classes of lake trout at each age as they have passed through the Opeongo fishery since 1936 are given in table 3. The values for the virtual populations in table 3 were obtained by using formula 1 (page 66) which is equivalent to summing diagonally the values for removal by ages given in table 2. For example, fish of the 1923 year class were captured at ages XIII to XVI in the years 1936 to 1939. No fish of this year class of age XVII were taken. The total catch of age XIII and older is $19 + 8 + 3 + 4 = 34$ fish and the virtual population at the beginning of age XIII is stated to be 34.

Lake trout enter the Opeongo fishery at age III and persist to age XVII, a period of fifteen years, so that with only 12 years' records it has not yet been possible to complete the observations on a single year class from the beginning of its entry into the fishery. However age groups XIII to XVII have in general been of minor importance and it is felt that the addition of the mean values of these groups where necessary to the 1931 to 1936 year classes has not introduced serious error.

TABLE 3

THE VIRTUAL POPULATION, $V(x;T)$, OF LAKE TROUT OF VARIOUS YEAR CLASSES T AT DIFFERENT AGES. BOLD FACE FIGURES ARE MEANS FOR THEIR COLUMNS

Year Class (Hatching Year)	Age x														
	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
$T - x$															
1919															11
1920														15	0
1921													10	0	0
1922												29	4	4	4
1923											34	15	7	4	0
1924										69	12	4	4	0	0
1925									140	22	18	15	0	0	0
1926								326	66	28	19	6	0	0	0
1927							685	207	31	28	4	4	4	4	4
1928						1396	731	176	86	39	33	22	20	20	16
1929					1861	1387	362	167	77	63	52	50	50	41	4
1930				1665	1432	782	343	120	74	63	54	48	26	10	3
1931			1371	1243	1045	625	232	112	90	82	58	49	86	6	**3**
1932		1294	1199	1165	890	569	205	101	73	73	42	21	14	**8**	
1933	1223	1193	1189	1062	841	407	207	154	112	68	31	20	**11**		
1934	1129	1122	1059	939	715	480	359	226	133	49	23	**20**			
1935	1277	1265	1229	1147	872	655	444	246	87	40	**28**				
1936	1388	1349	1265	1121	1004	732	530	157	64	**46**					

With the aid of the extrapolation mentioned above, estimates of the total contribution of four year classes (1933–1936) to the Opeongo lake trout fishery can be made. These are surprisingly small, varying from 1100 to 1400 fish, an average of less than one fish to ten acres of water surface. The contributions of the five preceding year classes, judging from their contributions since 1936, were somewhat larger, although probably no more than twice as great in any instance since the level of exploitation of the fishery was considerably lower before 1936 than it has been subsequent to that year.

Maximum Estimates of the Rate of Exploitation

The figures for the virtual population of lake trout at various ages given in table 3 represent minimum values for the number of fish at each age present in the lake at the beginning of a given fishing season, since all these fish which were of the age in question at that time were subsequently captured. Others of the same age were no doubt also present which subsequently died from causes other than the angler's troll.

An upper limit can therefore be set to the level of exploitation $K(x;T)/V(x;T) \times 100$ by calculating the percentage of the virtual

population removed in each fishing year. Let us take as an example the members of the 1930 year class removed in 1937, in which year they were age VII. In table 2 the estimate of the catch, $K(x;T)$, from this age group in 1937 is 650. In table 3 the estimate of the virtual population, $V(x;T)$, of this year class as of the beginning of year VII is 1432. The maximum value for the percentage of that year class which was removed in 1937 is therefore 45.4%.

Percentage removals were calculated in this manner for each value given for the virtual population in table 3 up to age XIII. These values were then averaged by age groups thus giving mean figures for the maximum level of exploitation. These mean values are plotted in figure 2 where they are referred to as the *virtual percentages captured*. It will be seen that these values are very low for ages III and IV, rise to a peak in the neighbourhood of ages VIII and IX and fall away again at the higher ages.

The Maximum Force of Fishing Mortality

The mean values for the maximum level of exploitation shown in figure 2 do not take the fishing effort into account. However if the premise be granted that for a given age group the yield per unit effort is proportional to the size of the population, then a maximum value can be derived for the force of fishing mortality by taking the virtual population to be the equivalent of the actual population. This assumes the natural mortality to be zero. In which case formula 11 (page 66) describes the decline of the population, and the maximum force of fishing mortality is obtained by inserting the appropriate numerical values in formula 11 (page 66). This force, $k(x)$max., is expressed here as the fraction of the population removed per 100 boat hours of fishing effort.

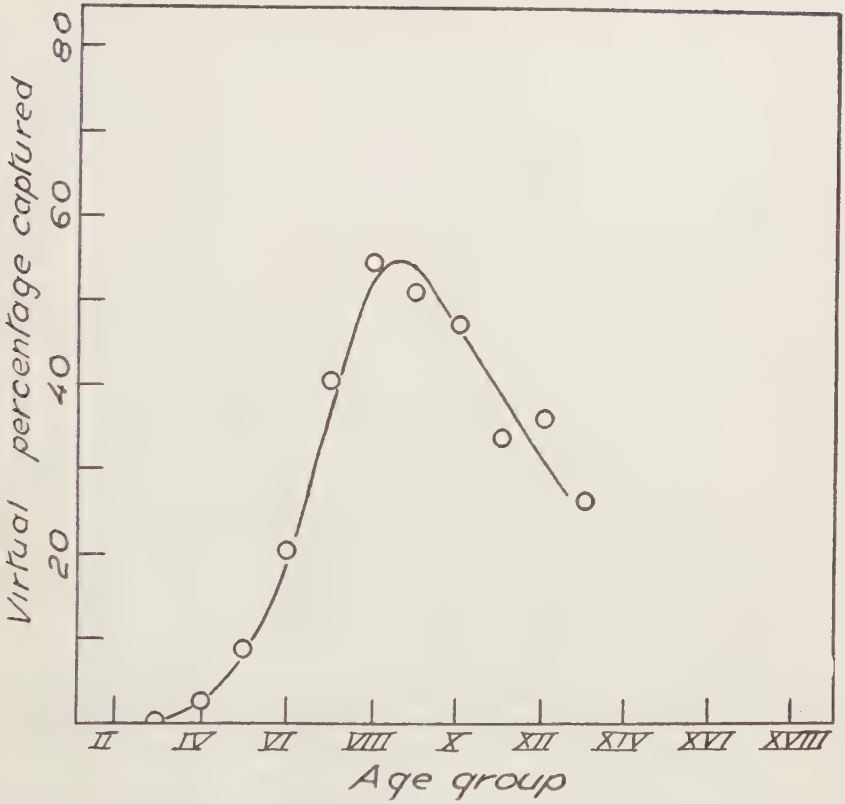
If we refer again to the example given above, the virtual percentage of 45.4% (0.454 when expressed as a fraction) was removed from the 1930 year class in 1937 by the expenditure of 22.4 units of effort. With the numerical values in the case cited, formula 11 becomes

$$1 - 0.454 = \exp\left[-22.4k(\mathrm{VII})\mathrm{max.}\right]$$

from which a value of 0.026 is obtained for k(VII)max. Values for k max calculated as above are given in table 4.

These values for the maximum estimate of the force of fishing mortality represent a mean response throughout the fishing season. The response of the population to the fishery is not all uniform within the period of the year when active fishing is being pursued, as is discussed

FIGURE 2.



THE PERCENTAGE OF THE VIRTUAL POPULATION CAPTURED AT DIFFERENT AGES OVER THE PERIOD 1937–1946. FOR A DEFINITION OF THE VIRTUAL POPULATION SEE PAGE 32. THESE VALUES REPRESENT MAXIMUM ESTIMATES FOR THE LEVEL OF EXPLOITATION OF THE OPEONGO LAKE TROUT POPULATION.

later (page 66). However, differences in fishing effort in different years are spread more or less uniformly over the whole season so changes in this mean value indicate changes in the condition of the fishery as a whole.

Removal of Year Classes

If the reduction in the number of members of a year class were completely known each year, it is obvious that plotting the cumulative loss in numbers against the cumulative percentage loss in terms of the original strength of entry would result in a straight line. The same is of

TABLE 4

MAXIMUM VALUES FOR THE FORCE OF FISHING MORTALITY, kx max., IN VARIOUS YEARS AT DIFFERENT AGES. THE VALUES REPRESENT MAXIMUM ESTIMATES OF THE MEAN FRACTION OF THE POPULATION REMOVED PER 100 BOAT HOURS OF FISHING EFFORT

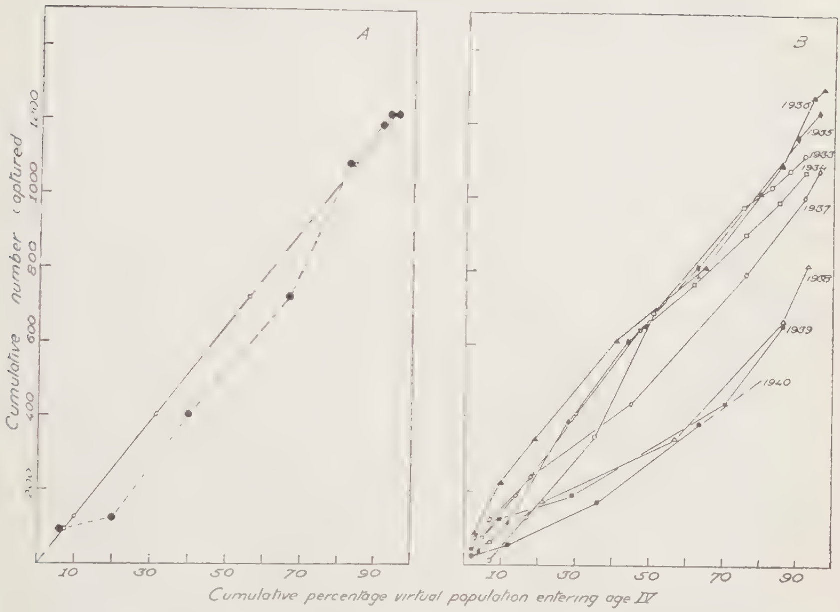
Year Class	IV	V	VI	VII	VIII	IX	X	XI
1926								.038
1927							.084	.006
1928						.063	.043	.057
1929					.060	.047	.056	.068
1930				.026	.050	.076	.042	.098
1931			.008	.031	.072	.061	.019	.021
1932		.0013	.017	.033	.087	.064	.061	.000
1933	.0004	.0071	.017	.062	.059	.050	.045	.054
1934	.0036	.0083	.023	.035	.050	.062	.059	.072
1935	.0022	.0063	.024	.050	.054	.065	.072	.045
1936	.0053	.0119	.018	.067	.034	.086	.051	.026
Mean	.0029	.0070	.018	.0435	.0582	.0638	.0532	.0486

course true if the catch and the virtual population are known, in which case ignorance concerning natural mortality is exactly balanced by ignorance concerning the precise value of the force of fishing mortality. A plot of the virtual population for the 1932 year class is shown as the unbroken line in figure 3A. If such straight lines could be plotted before the actual size of the virtual population had been known, or in other words before the year class had passed completely through the fishery, they would have a prediction value since they could be extrapolated to give an estimate of the probable total catch from that year class.

Curves of this type can be constructed by the use of the mean values of k max and the effort expended in the various years by again using formula 11 to determine the probable percentage of the virtual population taken in the year in question. This estimate will be fictitious in that it will not take into account the actual degree of catchability of the fish in that particular year, since a mean value will be used for the force of fishing mortality. However, it is assumed that over a few consecutive years fluctuations in catchability will be in both directions from the mean and that if such an estimate is low in one year, the estimate for the same year class another year may be equally high.

The broken line in figure 3A is a replot of the 1932 year class using the mean values for k max. The degree of deviation from linearity is

FIGURE 3.



THE RELATION OF EXPLOITATION TO REMOVAL IN VARIOUS YEAR CLASSES.
(For explanation see text.)

considerable but an estimate based on the four lower points would only give an underestimate of 20% for the final catch.

Other curves in which the mean value of k max were used to calculate the probable exploitation of the virtual population are presented in figure 3B. Arithmetically the derivation of the estimated percentages is a little round-about but not complicated. As an example let us take the first two terms for the 1932 year class. Fish of that year class were captured at age IV in 1936 and age V in 1937. From table 1 $E(1936) = 20.3$ and $E(1937) = 22.4$. From table 4 the means for the maximum force of fishing mortality are k max (IV) = 0.0029 and k max (V) = 0.0070. Therefore, the values for exp $(-k \text{ max } E)$ for these two age groups from the 1932 year class are respectively 0.059 and 0.157, which yield values for $1 - K(x;T)/V(x;T)$ of .94 and .85. The value of .94 for $1 - K(x;T)/V(x;T)$ at age IV gives an estimate of 6% for the percentage removed from the virtual population attaining that age. Similarly the estimate of removal from the virtual population attainingage V is 15%; and since only 94% of those attaining age IV were estimated to attain

age V, then the further removal from the virtual population entering the fishery was $(15 \times 94)/100 = 14\%$. And the catch for ages IV and V represents a cumulative removal of 20% of the virtual population attaining age IV.

The points for a given year class shown in figure 3B scatter considerably and the trend is certainly not at all stabilized until about 50% of the potential catch has been removed and the year classes which have gone completely through the fishery did not all attain their final rank until the removal had been 80%. Thus as a means of prediction, these removal curves lack somewhat in precision. However, they distinguish between the best and worst year classes at about 40% removal. Moreover, it is also probable that the curve for the 1932 year class, which is the most out of order, sags at its lower end because in the years of the fishery when members of this year class were young, such small fish were often eaten on the beach instead of being recorded in the creel census.

Estimates of the probable rank of the year classes shown in figure 3, based on the catch to 80% removal, are given in table 11 where they are all compared with the 1938 year class. The yields from the earlier year classes measured were about one and two-thirds that of the 1937 year class. The estimate for the 1940 year class is about 80% of the yield of the year class hatched in 1938.

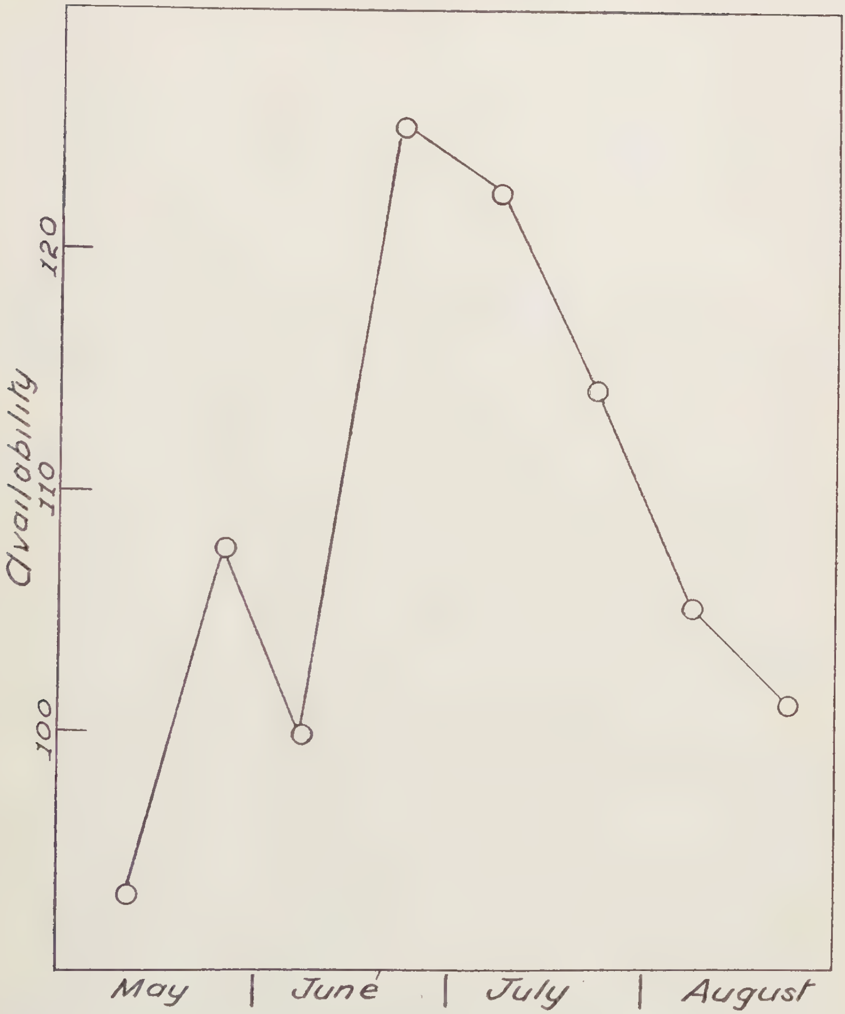
STATISTICS OF AVAILABILITY

Introduction

The yield per unit effort, $C(T)$, or availability of lake trout in lake Opeongo has varied from 128 fish per 100 boat hours to a low of 70. There is typically a seasonal cycle in the availability which is illustrated in figure 4. Typically lake trout are hard to catch in early May. They become somewhat more available in late May but there is a recession in fishing success in early June. Fishing is at its best in late June and early July. Subsequently fishing falls off progressively in late July and August. The trend in fishing in September is not shown in figure 4 since data were lacking for some of the years, but in general it improves somewhat over the August availability. Early October fishing is usually extremely poor. The season closes about October 15. This annual cycle in availability has been related to the migratory and feeding behaviour of the lake trout in response to the summer cycle of thermal stratification (Fry 1939[b]).

Within a fishing season, therefore, there is no simple relation between the size of the population and the yield per unit effort. This is not sur-

FIGURE 4.



THE ANNUAL CYCLE OF AVAILABILITY OF LAKE TROUT IN LAKE OPEONGO, ONTARIO. THE CURVE IS BASED ON THE MEANS FOR THE YEARS 1936 TO 1947. AVAILABILITY IS EXPRESSED AS THE NUMBER OF LAKE TROUT CAPTURED PER 100 BOAT HOURS.

prising since the fishing takes place in those months when thermal conditions in the lake are changing continuously and are having a marked effect on the activity and distribution of the fish.

While the curve shown in figure 4 is the mean for the 10 year period it is not inevitable that the cycle take this form every year. Apparently

annual variations in hydrological conditions in lake Opeongo offer enough variety to make the annual response of the lake trout vary.

The Relation of Age to Availability

The availability of lake trout of different ages in lake Opeongo has been determined by proportioning the yield per 100 boat hours according to the age composition of the catch. These data are presented in table 5.

TABLE 5

MEAN ANNUAL VALUES OF AVAILABILITY, $C(T)$, OF LAKE TROUT OF DIFFERENT AGES IN LAKE OPEONGO IN THE YEARS 1937-1947, EXPRESSED AS THE NUMBER CAPTURED PER 100 BOAT HOURS. FOR ANNUAL TOTALS SEE TABLE 1.

Year of Capture	Age														
	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
1937	0	0.2	1.5	8.9	28.1	45.9	24.8	7.9	1.7	0.2	0.3	0.3	0	0	0
1938	0.8	4.5	7.2	16.8	25.6	26.9	11.9	5.5	0.2	0.6	0.2	0	0.2	0	0
1939	2.8	2.6	8.2	15.7	22.9	28.0	15.9	6.4	3.3	1.7	0.9	1.1	0.3	0.3	0.3
1940	1.7	7.2	7.0	19.2	37.2	31.2	10.3	3.9	1.2	0.5	0	0.5	0	0	0
1941	0.8	7.0	12.7	24.1	20.6	17.7	9.2	2.0	0.1	0.1	0.1	0	0	0	0
1942	0.2	3.1	8.2	21.0	38.2	21.4	9.3	5.1	1.2	1.6	0.1	0.1	0	0	0
1943	0.8	5.9	5.9	17.6	37.8	29.4	18.5	5.9	0	3.4	0.8	0	0	0	0
1944	1.1	3.2	10.1	13.8	23.4	24.4	23.9	11.1	5.3	3.7	1.1	2.7	1.1	0.5	0.5
1945	0	0.7	2.2	4.7	15.3	24.1	25.5	10.9	5.8	2.6	1.5	2.9	1.1	2.6	1.1
1946	1.1	1.7	4.5	6.7	13.8	18.7	12.4	5.4	2.8	1.5	0.6	0.4	0	0.4	0.2
1947	0.2	2.5	4.4	6.9	17.5	17.8	12.4	6.2	1.5	1.0	0.2	0	0.5	0.2	0.2

The availability of lake trout in lake Opeongo is highest at age VII, which is two years before the age at which k max is highest. The availability at age VIII is, however, almost equally high. Availability at the higher ages drops rapidly, presumably due to the drop in the size of the population.

The prime question with regard to availability is whether it bears any relation to the size of the population fished and what that relation may be. It has already been demonstrated (figure 4) that within a fishing season the relation of availability to the population must change. It remains to be seen whether the annual means have any simpler relation. The relation between the annual mean of availability at various ages to the virtual population at the same age is shown in figure 5.

As figure 5 indicates there are varying degrees of correlation between availability and the virtual population which range from none at all at age VI, since the negative trend displayed by these points can be considered absurd, to highly significant correlations at higher ages.

FIGURE 5.

THE RELATION BETWEEN AVAILABILITY AND THE SIZE OF THE VIRTUAL POPULATION AT VARIOUS AGES. NOTE THAT THE MEAN VIRTUAL POPULATION WITHIN THE SEASON IS USED, THE VIRTUAL POPULATIONS GIVEN IN TABLE 3 ARE ESTIMATES FOR THE BEGINNING OF THE FISHING SEASON. THE LINES TO INDICATE THE TRENDS WERE FITTED BY EYE.

Lack of correlation between availability and the size of the virtual population does not necessarily mean similar lack of correlation between availability and the true population, for the difficulty may be major fluctuations in natural mortality which would tend to make the values for the virtual population the more erratic of the two values correlated. However, the seasonal changes in availability shown in figure 4 cannot all be due to mortality. This conclusion is particularly strengthened by the fact that the lake trout in lake Opeongo undertake a summer migration which seems almost bound to influence their catchability as indeed all the facts known about their response demonstrate (Fry 1939[b]). There is also some indication that in a given year the trend in the relation between availability and the size of the virtual population is in the same direction in both ages VI and VII. The years of capture are indi-

cated on the graph for the various points. The correlation, however, is not strong and more data are required before it can be considered that the trend is definite. It appears reasonable to conclude, therefore, that lack of correlation between availability and the virtual population is more likely to be due to fluctuations in the annual migratory cycle than to major fluctuations in natural mortality although this latter possibility cannot be entirely disregarded.

In those cases where there is a close correlation between availability and the virtual population no such problem of interpretation exists. It may be concluded with confidence that in these cases both the availability and the virtual population present stable reflections of the true population. Since the annual cycle of availability is exhibited by these older fish as well as the younger ones, it is to be presumed that the older fish present a more stable response to the annual migratory cycle and thus a more consistent annual cycle of availability.

A second interpretation that might suggest itself is that, since the size of fish appears to have a strong influence on their availability, annual differences in growth rate might have a marked influence on the availability of the entering age groups. This, however, does not appear to follow from our data as a comparison of tables 5 and 7 will demonstrate.

STATISTICS OF GROWTH

Introduction

The introduction of the method of making scale impressions by passing a strip of cellulose acetate with the scales outer side down on it through a jeweler's roller has greatly facilitated the preparation of lake trout scales for reading. We learned of this method through W. R. Martin of the Atlantic Biological Station. Our impressions were made on cellulose acetate 0.040 inches thick without heating. No cleaning of the scales was necessary. Before the scale impression method was adopted the scales were cleaned and mounted in glycerine waterglass.

Three to four scales were prepared and one read, the rest not being examined unless difficulty was encountered in the first one. All places where a complete circulus succeeded a series of incomplete ones were accepted as annuli. Reading of the scales was begun in 1938 but by far the greater number were read in 1947. I am indebted to Miss L. C. Craigie and F. P. Maher for most of the scale readings.

Growth in Length

Although the significant dimension from the point of view of produc-

tion is weight, the size age relationship was worked out primarily from measurements of body length. This course was adopted because of the greater ease with which measurements of length could be made in the field and especially because the fish were often dressed before the census worker had an opportunity of examining them. However, extensive series of weight determinations were also made so that it is believed that there is but little error in the age weight relationships deduced from the age length data.

To determine the degree of variation of the growth rate in the various basins, readings for fish of the VI, VII and VIII groups were separated into locality in cases where the place of capture had been noted. The means for these data are given in table 6. No substantial difference was found in the size attained at these three ages in any of the four basins. It is assumed that growth would also be similar at other ages.

TABLE 6

SIZE COMPOSITION OF CATCHES OF AGE GROUPS VI, VII AND VIII TAKEN IN THE YEARS 1937, 38, 39, 41, 42, 43, 44, 45 AND 46 ARRANGED ACCORDING TO LOCALITY OF CAPTURE

Age	Locality	Fork length inches										Mean
		14	15	16	17	18	19	20	21	22	23	Length Inches
VI	North Arm	3	9	26	10	2						16.0
	South Arm	4	14	25	19	9	4					16.4
	East Arm		19	37	38	15	5					16.6
VII	North Arm			14	39	42	20	6	2	1		17.8
	South Arm			18	38	51	29	3	1	0	1	17.9
	East Arm			10	28	27	27	0	2	2		17.9
	Annie Bay			5	15	19	12					17.7
VIII	North Arm			2	6	43	40	29	11	1	1	19.0
	South Arm			6	20	42	57	25	2			18.5
	East Arm			3	11	43	46	24	2	2	1	18.8
	Annie Bay				4	11	13	4	2			18.7

There has been but little variation in the average size reached at a given age in the different years as table 7 indicates. What variation there has been does not appear to show any particular trend and would seem to reflect no more than annual variations in growing conditions

TABLE 7

THE EFFECT OF CALENDAR YEAR AND SEX ON THE AVERAGE LENGTHS ATTAINED BY LAKE TROUT OF LAKE OPEONGO AT VARIOUS AGES

Year of Capture	Age					
	V	VI	VII	VIII	IX	X
1937	—	16.7	18.1	19.3	20.6	21.9
1938	14.1	16.7	17.9	19.2	20.1	21.8
1939	14.7	15.9	17.7	18.7	20.1	21.6
1940	14.6	16.3	17.9	19.2	20.2	22.3
1941	14.7	16.3	17.5	18.9	20.4	22.1
1942	14.7	16.8	18.3	19.1	20.2	21.2
1943	14.9	16.4	17.8	19.0	20.4	22.8
1944	14.7	16.7	18.2	18.8	19.8	20.6
1945	14.5	15.9	17.3	18.7	19.9	21.2
1946	14.5	16.0	17.5	18.6	19.7	21.6
1947	14.0	16.0	17.5	18.5	21.1	—
1943 to 1947 ♂	14.7	16.4	17.7	18.6	19.9	21.6
1943 to 1947 ♀	14.7	16.2	17.6	18.8	20.3	21.2

There is no evidence of any progressive change in growth rate attendant on the increased exploitation of the fishery which has occurred since the opening of the Algonquin Park highway in 1936. It also appears that there is not appreciable difference in the growth rate of the two sexes. Table 7 shows a comparison of the average lengths attained by the two sexes at various ages. The relation between length and age found for all samples from 1937 to 1946 inclusive are shown in table 8.

The growth in length of the Opeongo lake trout as determined by our scale readings is considerably slower than had been inferred from the modes found in a sample gillnetted in 1936 (Fry and Kennedy 1937). The growth rate of the Opeongo fish is most similar to that reported by Juday and Schneberger (1930) for Wisconsin lakes being less than that found in the populations studied by Applegate 1947, Van Oosten 1943 and Royce M.S. The growth rate in Opeongo is, however, considerably higher than that reported for the lakes of the North West Territories. (Various authors 1947, Miller and Kennedy 1948).

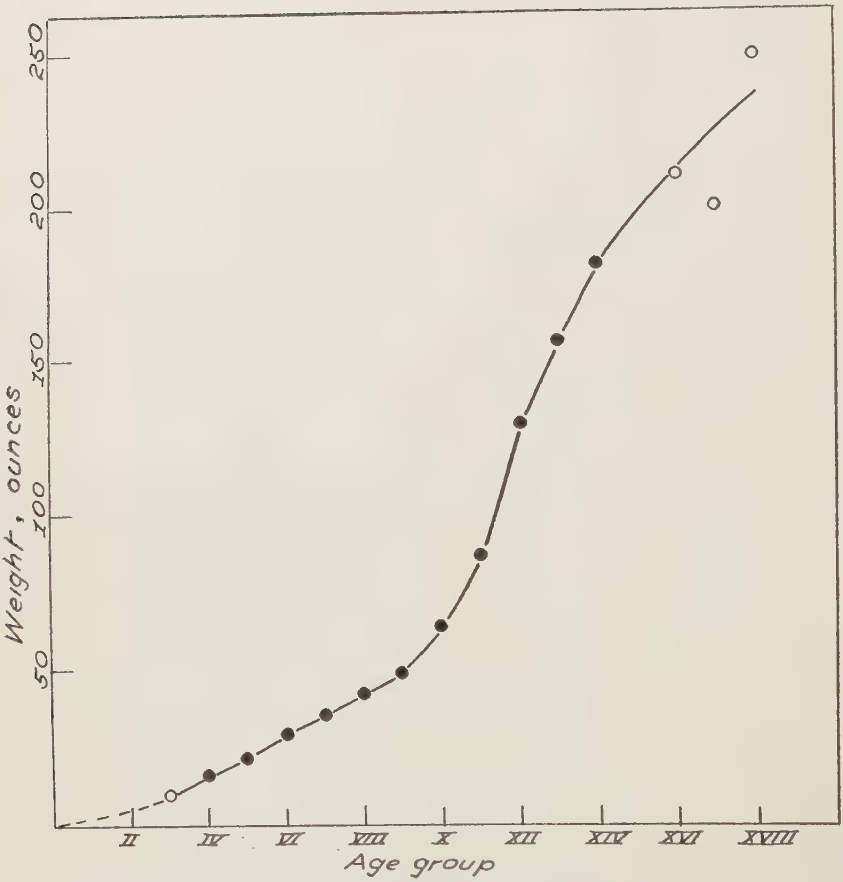
It is rather disappointing that in spite of the large series of scales, no clearcut growth trend within the year has been evident. This may perhaps be because there are fewer samples for the months of May and

TABLE 8

AGE-LENGTH RELATIONSHIP OF LAKE TROUT CAUGHT IN LAKE OPEONGO FROM 1937 TO 1946 INCLUSIVE

Age	Length class inches																										
	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
III	1	5	14	20	6																						
IV		2	4	23	51	57	6																				
V			5	5	19	66	132	25	7																		
VI						19	99	222	169	69	14	1	1														
VII			1	0	0	0	4	86	304	355	238	34	16	2	0	0	0	0	1	1							
VIII					1	0	1	23	89	311	353	217	96	37	3	3	0	0	0	0	0	0	0	1			
IX									3	62	113	232	154	51	12	2	1	0	0	0	1						
X								1	0	0	8	51	64	62	28	19	5	2	3	1							
XI											1	5	12	14	27	10	5	2	1	5	2	2					
XII														2	5	8	9	9	10	5	1	0	1	0	0	1	
XIII																1	2	7	1	6	4	1	1	1			
XIV															1	3	2	1	5	6	2	1	1	1	1	1	2
XV																			1	1	1	0	3	2			
XVI																		1	1	0	3	2	5				
XVII																					1	1	1	2	2		
Mean weight for length class, ounces	6.0	6.8	8.8	10.4	14.6	18.8	23.8	27.4	31.9	36.6	41.0	48.1	54.3	63.5	79.0	98.2	120	133	146	154	187	197	234	250	312	312	313

FIGURE 6.



THE AGE-WEIGHT RELATIONSHIP OF OPEONGO LAKE TROUT BASED ON SCALE SAMPLES TAKEN FROM ANGLERS' CATCHES 1937 TO 1945.

June which are possibly the months of growth. There is also the possibility that selection by the fishery operates to take the fish of the entering classes as they reach a certain size. This would tend to suppress any evidence of seasonal growth since the more rapidly growing ones would be taken earliest in the season and those growing more slowly later on

Growth in Weight

The mean age weight relationship is shown in figure 6. The mean weight for a given age was derived by referring the appropriate weight

for each length class in table 8 to the length distributions and taking the means.

The Opeongo lake trout grow slowly in weight for an extended period of years. The increase is almost linear from year II to year IX with an average increment of six ounces per year. About year IX the rate increases, the increment being greatest between X and XI but continuing to be rapid up to age XVII, the age of the oldest fish captured. The course of the growth curve for the Opeongo trout is correlated with the size of the food they take. The younger age groups feed on small perch and whitefish, the older fish eat larger whitefish and large suckers. The period of slow growth occurs during the years when there is a transition in feeding habits. The growth curve as presented is biased of course by selection due to the fishing method which probably takes more of the faster growing members of the younger groups and tends to flatten the lower portion of the curve.

The average weight attained at each age and the increments between the ages are given in table 9. These mean data have been used in all calculations here that relate to weight.

TABLE 9

AGE-SIZE RELATIONSHIP OF OPEONGO LAKE TROUT

Age Group	Average length (ins)	Average weight (ozs)	Weight Increment (ozs)	Number of specimens
III	11.5	9.9	—	46
IV	13.2	15.7	5.8	143
V	14.6	21.8	6.1	268
VI	16.4	29.3	7.5	294
VII	17.9	36.3	7.0	1042
VIII	19.0	42.5	6.2	1135
IX	20.1	49.5	7.0	631
X	21.6	64.6	15.1	244
XI	23.3	87.0	22.4	86
XII	25.8	129	42	51
XIII	27.5	157	28	24
XIV	30.3	182	25	27
XV	30.5	211	29	8
XVI	29.6	200	24*	12
XVII	31.4	249	24*	7

*½ increment XVI-XVII.

Minimum Production of Lake Trout in Lake Opeongo

Estimates of the minimum increment of weight of the lake trout population have been made for the years 1936, 1937 and 1938 by combining the growth data with the estimates of virtual populations. These calculations have been made as follows.

For each age group the size of the virtual population at the beginning of the fishing season following the year for which the calculation was desired, was taken from table 3. Thus if the production were being calculated for 1936 the virtual population for 1937 would be taken. All these fish would have been present in the lake throughout the previous year and their increment in weight in that season represents a production of lake trout in that season. In addition the individuals captured during the season in question also make some growth in the current year before they are taken.

Since the seasonal distribution of growth of the Opeongo lake trout is not known, a direct calculation of the amount of growth made in the current season is not possible. However, the peak of the Opeongo fishery is in July, about one-third of the total catch being taken in this month. Another third are taken in August and September. Thus it may be expected, if the seasonal growth pattern of the lake trout is similar to that of other salmonids, for example the whitefish (Kennedy 1943), the current season's growth of over half the fish taken will have been completed before they are captured. Most of the remainder would have also grown to some extent. Therefore, it has been assumed that about 75% of the current year's increment would have been attained by the catch as a whole. Hence, for the purpose of estimating production, 75% of the current season's catch from table 2 have been added to the virtual population at the end of the season. This correction for the current season's growth is probably somewhat high but such an error is balanced to a certain extent by the fact that the virtual population must be smaller than the true population.

Finally, since the three youngest age groups have not been sampled and the increments for them are not known, the total increment to age III was calculated for the virtual population in that age group.

The minimum estimates made in this manner for the three years were: 1936, 6200 lbs.; 1937, 5100 lbs. and 1938, 4040 lbs. The estimated removals in these years were respectively 9400, 7450 and 3940 lbs. In 1938 the removal and the minimum production were equal and it would appear that the fishery in that year was not a drain on the somatoplasm of the trout population. In 1936 and 1937 the removal was

somewhat greater than the minimum estimates of production, but not twice as great. Since natural mortality is not known and fish living in those years which subsequently died without being accounted for in the virtual population would have added to the production at that time, it is likely that from the point of view of removal of trout flesh the drain on the fishery was not excessive in those years either.

STATISTICS OF REPRODUCTION

Introduction

The sex ratio of the lake trout in lake Opeongo is almost perfectly 50–50. The observed percentage of females in 2774 lake trout examined over the years 1937 to 1941 was 50.3%. As usual among fish the males mature slightly in advance of the females. The relation of age to maturity in the Opeongo lake trout is similar to that reported elsewhere (Surber 1933, Royce M.S. 1943). The first male lake trout mature in lake Opeongo in year IV and all appear to be mature by year VII. While there appear to be a few females which also mature in year IV these are rare, and in general the age curve for onset of maturity in females lags a year behind that for males.

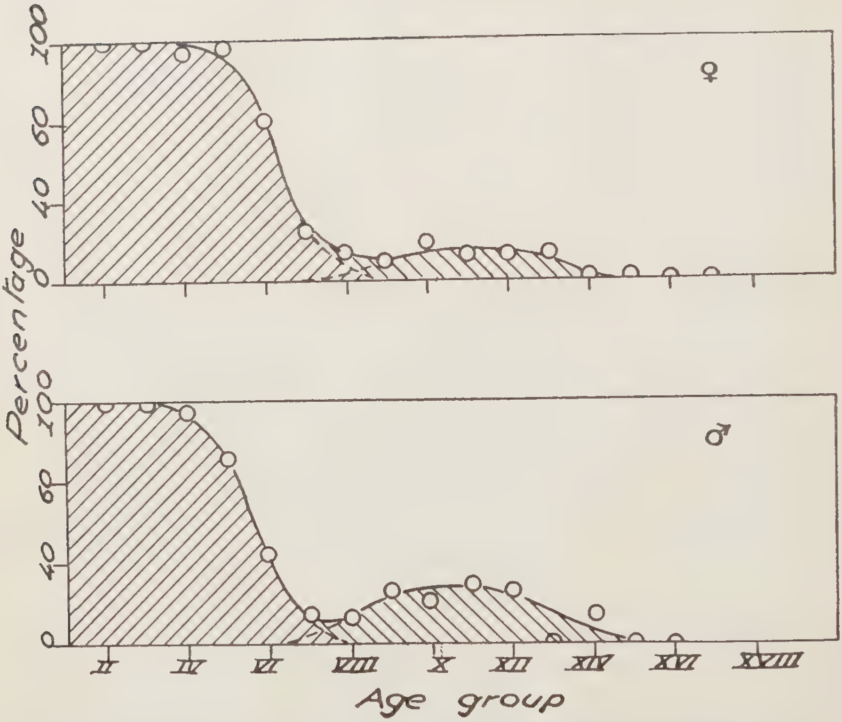
At some period after they reach maturity it is not unusual for Opeongo lake trout of both sexes to fail to produce gametes in a particular season. Such fish have been termed infertile. There appears to be no permanent damage to the gonads and it is probable that the fish spawn again in later years.

The relation between fecundity and age in Opeongo lake trout is shown in figure 7. The hatched portions show the percentage of infecund fish in each age group. Up to age VIII this hatched portion undoubtedly represents immature fish. Beyond that age it is probable that the infecund fraction of the population is made up of infertile individuals rather than of fish of delayed maturity. In the case of females this is certain since the ovary of a lake trout which has never spawned can be distinguished from that of one which has previously shed eggs.

Egg Count in Relation of Age

Estimates of the number of eggs were made on all mature females whose viscera were available and in which the eggs maturing in the current season were over 2 mm. in diameter. Except for estimates in 1937 these counts were made by measuring the diameters of 10 of the maturing eggs and weighing the ovaries. The count was determined by

FIGURE 7.



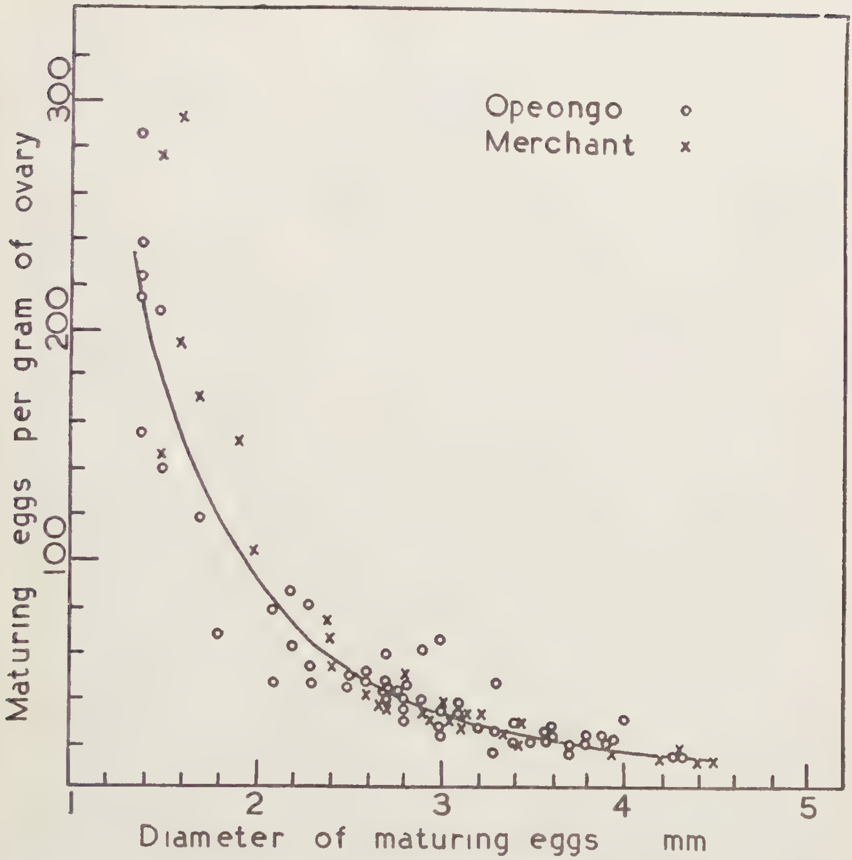
THE RELATION BETWEEN AGE AND FECUNDITY IN LAKE TROUT TAKEN BY ANGLING. THE ORDINATE IS PERCENTAGE INFECUND, AVERAGE VALUES 1937 TO 1941.

using the conversion diagram given in figure 8 which it is believed holds good for lake trout in general. The egg diameters were obtained by dissecting the eggs from the stroma and measuring 10 of them placed in line together.

The average egg count in relation to age is shown in figure 9. The average number of eggs carried by an age V female is about 1200. The mean number increases along a smooth curve to 6100 at age XIII. Beyond that age the data are scanty but are grouped about an extrapolation of the same curve. The estimated mean for females of age XVII is 15,000 eggs.

A feature to be noted in figure 9 is that in a given year the egg count of fish of all ages may be consistently higher or lower than the five year mean. Thus the 1938 counts are higher and the 1943 counts are lower than the mean. This suggests that dietary conditions at the time the

FIGURE 8.

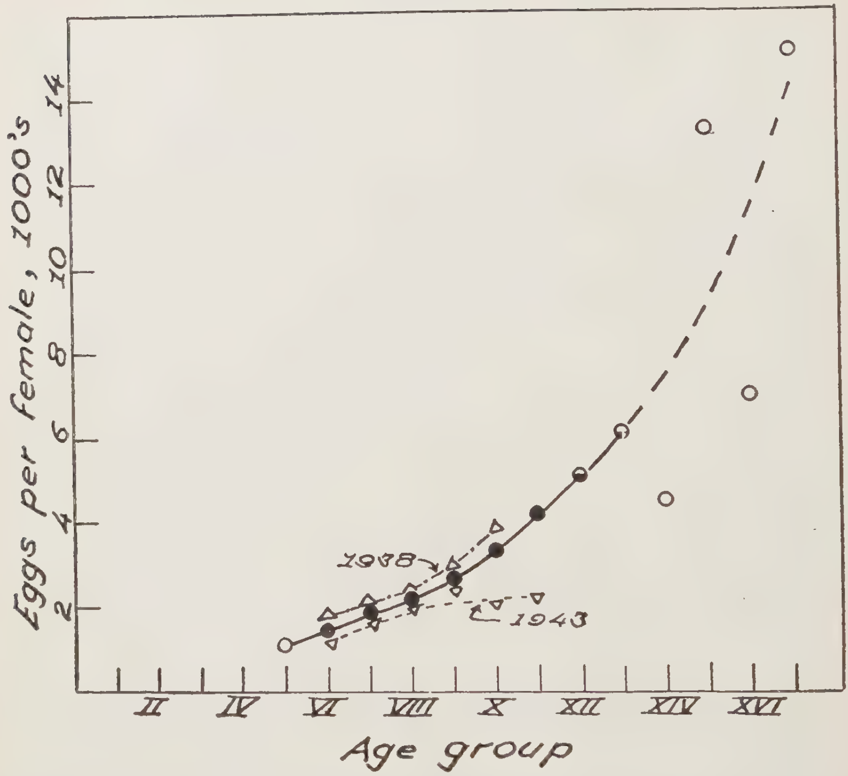


THE RELATION BETWEEN EGG DIAMETER AND NUMBER OF MATURING EGGS PER GRAM OF OVARY IN LAKE TROUT FROM OPEONGO AND MERCHANT LAKES, ALGONQUIN PARK.

eggs are first laid down, probably two years previous to the one in which they mature, influence the number of eggs produced. Two seasons in which there was a marked difference in the availability of perch to the Opeongo lake trout are given in Fry (1939[a]) figure 1.

Table 10 which gives the average number of eggs per individual for each age group supplies the final fecundity data used in calculating the spawning escapements given below. This table was constructed by combining the egg count per fecund female (figure 9) with the percentage of fecund females in each age group.

FIGURE 9.



EGG COUNT IN RELATION TO AGE IN OPEONGO LAKE TROUT. THE MEAN CURVE DRAWN IS BASED ON EGG COUNTS FOR THE YEARS 1937–1941 INCLUSIVE.

TABLE 10

AVERAGE NUMBER OF MATURING EGGS PER INDIVIDUAL $m(x)$ IN LAKE TROUT OF VARIOUS AGES CAPTURED IN LAKE OPEONGO IN SUMMER BASED ON EXAMINATION OF FISH TAKEN BY ANGLING IN THE YEARS 1937 TO 1941 INCLUSIVE.

Age	Eggs per individual	Age	Eggs per individual
IV	0	XI	1750
V	58	XII	2130
VI	294	XIII	2740
VII	700	XIV	3800
VIII	1030	XV	5220
IX	1270	XVI	7840
X	1500	XVII	11900

Because of the annual fluctuation in the egg count age relationship, and probably also in the percentage infertility, the application of such mean fecundity data offers the possibility of appreciable error. However, this course has been chosen because of its statistical convenience and because of the paucity of egg count data in some seasons.

Spawning Escapement

Two partially independent estimates of the spawning escapement of the Opeongo lake trout population can be made. One of these, the *minimum spawning escapement*, has been made from the size of the virtual population and its age distribution combined with the fecundity data. The other, the *relative spawning escapement*, is based on the magnitude of the egg loss in the season preceding the spawning period and the magnitude of the fishing effort which brings about this loss of eggs. The minimum spawning escapement is an estimate of absolute magnitude whereas the relative spawning escapement can only give a ratio between two years.

Minimum Spawning Escapement. The data from which the minimum spawning escapement is calculated are contained in tables 3 and 10. Table 3 gives the virtual populations estimated to be present in the years of the investigation. As was discussed previously these values are the estimates of the size of the virtual population present at the beginning of each fishing season. Thus at least 1223 members of the 1933 year class were present at the beginning of the fishing season in which they attained age III (1936) and so on. To calculate the minimum spawning escapement these values for the beginning of a fishing season have to be referred back to the end of the season previous to it. Thus the 1223 lake trout of age III present at the beginning of the 1936 season would be considered as 1223 individuals of age II at the end of the 1935 season.

For the purpose of calculating the minimum spawning escapement in 1935 the virtual population of age groups VI to XVII at the beginning of the 1936 season were used. The number of fish in each age group within this range was multiplied by the average number of eggs per individual assigned to the age group immediately below it. Thus the 1665 fish of age VI estimated to be present in the virtual population was multiplied by 58, the average number of eggs for age V, and so on, this being the number of eggs these individuals could have been expected to have deposited the previous fall.

The minimum spawning escapement can be calculated for the spawning years 1935 to 1940 from the removal data available to the end of

1947. These values contain the averages for the older age classes mentioned on page 54. These estimates are given in table 11 together with estimates for the relative spawning escapement.

TABLE 11

ESTIMATES OF SPAWNING ESCAPEMENT AND YEAR CLASS STRENGTH. THE RATIOS FOR COMPARISON ARE ALL BASED ON TAKING THE 1937 SPAWNING AND THE RESULTING ESCAPEMENT, THE 1938 YEAR CLASS, AS UNITY. THE RELATIVE YEAR CLASS STRENGTH WAS ESTIMATED FROM THE NUMBER REMOVED WHICH REPRESENTED 80% OF THE VIRTUAL POPULATION ENTERING AGE 4. THESE VALUES WERE READ FROM FIGURE 3.

Spawning year	Min. Spawn Escapement		Egg loss millions	Rel. Spawn Escapement ratio to 1937	Relative year class strength
	eggs X 10^6	ratio to 1937			
1931					1.7
1932					1.7
1933					1.6
1934					1.7
1935	3.43	1.99			1.4
1936	2.70	1.57	2.86	1.21	1.0
1937	1.71	1.00	2.67	1.00	1.0
1938	1.65	0.96	1.20	0.69	0.8
1939	1.30	0.76	1.44	0.89	
1940	1.22	0.71	1.03	0.76	
1941			0.70	0.56	
1942			0.47	0.79	
1943			0.74	0.97	
1944			1.15	1.14	
1945			2.23	1.40	
1946			1.25	0.68	
1947			0.92	0.66	

Relative Spawning Escapement. The lag of the estimate of the minimum spawning escapement behind the fishery is a serious limitation to its worth. It can be used only as a research tool to check the event long after its occurrence. This limitation does not restrict the use of the second estimate, the *relative spawning escapement*. The relative spawning escapement gives an estimate for the current season. The calculations for estimating the relative spawning escapement were kindly worked out for me some years ago by Dr. D. B. DeLury of the Ontario Research Foundation. They are based on his general formula (DeLury 1947) recently published.

The problem is essentially that of determining the size of the population at the end of a given fishing season and then making an estimate of the spawning potential of these survivors from a knowledge of the egg count. Since the actual values for the numbers dying naturally, $R(x;T)$, and for the fraction of the population taken per unit of fishing effort, $k(x;T)$, are not known for any age, an estimate of the absolute quantity of spawn cannot be attempted. However, the ratio of the spawning escapement in two years is not greatly affected by moderate variations in these constants so that the relative spawning may be calculated with some assurance.

The assumption is made that there are no changes in the population of eggs during the fishing season apart from loss in the fish captured. This assumption specifies what Ricker (1940) defines as a fishery of Type I. It is entirely justifiable from the point of view of recruitment since the number of lake trout eggs which are going to mature in a given season is fixed at least the year previous. Natural mortality has been considered zero; this is compensated in part by using k max for the force of fishing mortality.

The appropriate formula for calculating the relative spawning escapement is developed in the appendix as equation 14. The computations were carried out as follows:

a. A value of 0.155 was assigned to the average of $k(x;T)$; this corresponds to a level of exploitation of 30% per year over all the age groups. This value is approximately the 10 year average for the maximum level of exploitation.
b. The total egg loss for a given year, $\sum m(x;T)\ K(x;T)$, was computed by summing the products of numbers of each age caught (table 2) by the mean number of eggs per individual for that age (table 10). Egg losses thus calculated are given in table 11.
c. The appropriate values for the units of fishing effort in the years compared were taken from table 1.
d. These numerical values were then employed in equation 14. To calculate the escapement in 1938 as compared with 1937 the values in the equation would be

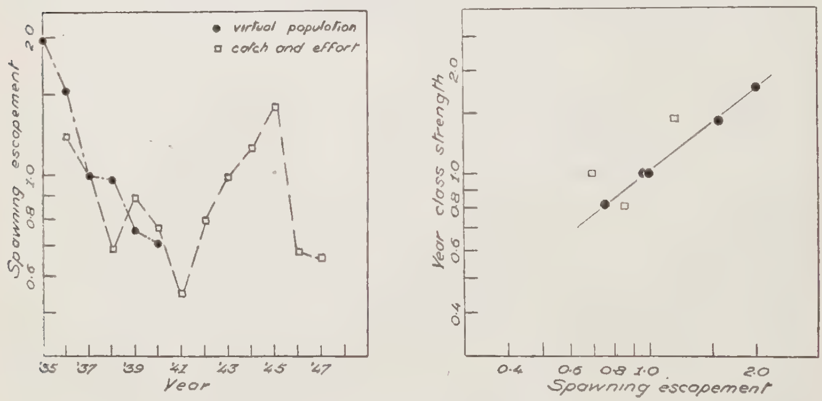
$$S(1938,\ 1937) = \frac{\text{egg loss 1938 } (\exp\{-0.155 \times 22.4\} - 1)}{\text{egg loss 1937 } (\exp\{-0.155 \times 16.3\} - 1)}$$

$$= \frac{1.3 \times 10^6 \times .410}{2.7 \times 10^6 \times .252} = 0.69.$$

DISCUSSION

The trends in the minimum and the relative spawning escapements are shown in the right hand panel in figure 10. The general agreement between the estimates made by the two methods may perhaps be considered to be good, although as yet they can be compared over only four years. However, there is a reversal in the estimates for the years 1938

FIGURE 10.



CHANGES IN THE ESTIMATED SPAWNING ESCAPEMENT IN THE LAKE TROUT POPULATION OF LAKE OPEONGO. THE SECOND PANEL IS THE RELATION BETWEEN THE ESTIMATED SPAWNING ESCAPEMENT AND THE ESTIMATED STRENGTH OF THE RESULTING YEAR CLASS. THE ESCAPEMENT OF 1937 AND THE HATCH OF 1938 ARE TAKEN AS UNITY. CIRCLES DENOTE ESTIMATES BASED ON THE MINIMUM SPAWNING ESCAPEMENT, SQUARES ESTIMATES BASED ON THE RELATIVE SPAWNING ESCAPEMENT.

and 1939. The estimates based on the minimum escapement show a reduction in spawning in 1939 as compared with 1938 while the estimate for the relative escapement shows the opposite. The reason for this discrepancy appears to lie in what appears to be the greatest weakness of the method by which the relative spawning escapement is derived. This weakness is the assumption that the catch per unit effort closely reflects the number of fish present. As has been pointed out earlier (page 32) there is not necessarily a close relation between the virtual population and the availability at ages VIII and younger. Unfortunately age groups V to VII make a mean contribution of about 50% to the total spawning escapement. Hence hydrological conditions which made these age groups more prone to capture in a given season would make the estimate of the relative spawning escapement considerably higher than the actual numerical strength of these age groups would

justify. The year 1940 in particular appears to have presented conditions favouring such circumstances.

On the whole, however, years of such exceptional hydrological conditions are probably rare and do not vitiate the general trend in relative spawning escapement shown in figure 10. This figure shows a steady drop in spawning escapement from 1935 to 1941, the first years following the opening of the Algonquin Park Highway. During this period spawning appears to have fallen to about one quarter of the 1935 level. With the reduced fishing intensity in lake Opeongo that was characteristic of the later war years the spawning escapement increased, rising again to approximately the 1936 level in 1945. Subsequently, increased fishing in 1946 and 1947 seems to have again reduced spawning escapement as sharply as it fell in the prewar years when access to lake Opeongo was first improved.

The possible effects of such changes in the size of the spawning escapement are not yet definitely established but unfortunately the only data at hand, that for the hatches of 1936 to 1941, indicate a strong positive correlation between year class strength and spawning escapement. This is illustrated in the second panel of figure 10. Consequently it is to be feared that the decline in spawning escapement which is the result of the increased level of exploitation of the lake trout population of lake Opeongo may materially reduce the strength of the entering year classes.

While too much reliance should not be placed on a correlation involving only five consecutive year classes, it must be pointed out that there is confirming evidence in comparative data. During this same period lake trout fishing in lake Opeongo has consistently fallen below the standard curve for the Algonquin park fisheries (Fry and Chapman 1948). This position in relation to the standard curve may be taken to indicate that the spawning in lake Opeongo may have been reduced to a level where it is limiting the size of the population.

Finally perhaps it should be pointed out that the relative spawning escapement can be estimated without any knowledge of the age of the fish concerned. Here the age-egg count relationship has been used but a size-egg count curve could be used instead. Indeed the earlier estimates for Opeongo were first made on that basis and were essentially the same as those made more recently on the basis of age composition.

NATURAL MORTALITY

The final, and indeed indispensable estimate required is that for the natural mortality occurring in the Opeongo lake trout population. A

maximum estimate of this has been attempted below by the application of DeLury's (1947) formula 5. DeLury pointed out that if the catch per unit effort bears a constant relation to the size of the population and if there is no natural mortality, then plotting the cumulative catch up to time t against the catch per unit effort at time t will result in a straight line. Deviations from such a straight line would indicate a change in the catchability of the organism or changes in the population not accounted for by the catch removed from it. Such changes in the population could result from recruitment or from natural mortality. If in constructing a curve of the type proposed by DeLury, allowance can be made for recruitment and changes in catchability, then deviations from a rectilinear relationship should give an estimate of the natural mortality involved.

Recruitment can be eliminated by considering fish of one year class only and following the removal and availability from year to year within this class.

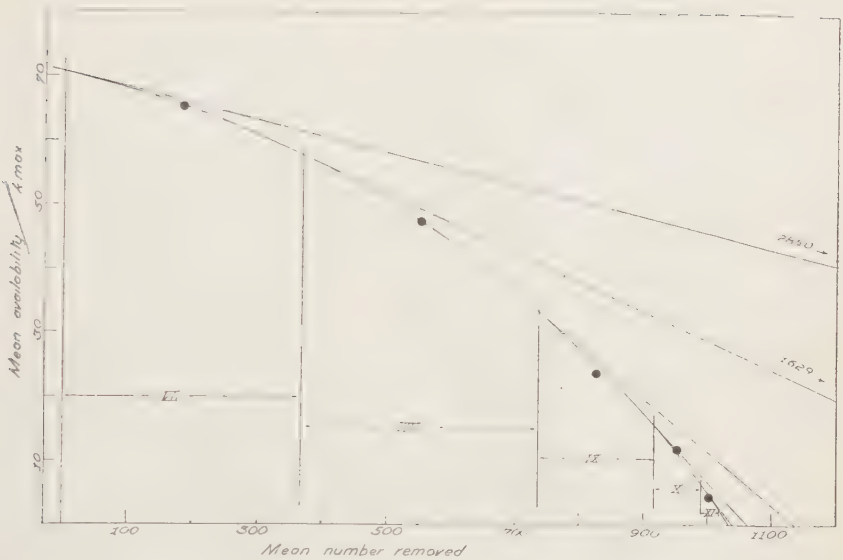
Owing to the lack of correspondence of availability with the size of the population within a fishing season the unit of time chosen has been the year. Differences between years due to annual differences in hydrological conditions can be smoothed by taking the means for each age of a series of year classes. Here year classes 1929 to 1937 have been averaged over ages VII to XI. The age groups have been restricted to this range since it is felt that the data are not yet extensive enough to warrant a consideration of all the age groups present in the fishery.

To compensate for the difference in catchability of fish of different ages the mean availability found at each age for the year classes in question has been divided by k max. Since k max is the highest possible value of the force of fishing mortality, the index of abundance derived in this manner must be a minimum value. Moreover, the younger the age the greater is the difference between k max and the true value for the force of fishing mortality. Hence the correction is more and more conservative as you proceed from the older to the younger year classes.

The curve drawn between the removal and these minimum indices of abundance is shown in figure 11. It is a curve sloping at an increasingly greater rate towards the abscissa and cutting it at a value which represents the virtual population at age VII.

If the values for the estimates of abundance had been precisely commensurate with the actual size of the population at each point considered, this curve would have been a straight line and its intercept on the abscissa would have indicated the size of the true population at

FIGURE 11.



THE CURVE USED IN MAKING THE MAXIMUM ESTIMATE OF NATURAL MORTALITY FROM AGES VII TO XI. FOR EXPLANATION SEE TEXT PAGE 58.

age VII instead of the virtual one. If the curve had been constructed by using the true force of fishing mortality as a divisor, then the intercept of a tangent drawn from the beginning of age VII to the abscissa would have indicated the true population at that age. The difference between the intercept of the curve (the virtual population) and the tangent (the true population) would represent the natural mortality in age VII and subsequent years.

A tangent drawn at the beginning of age VII to the curve in figure 11 has a lesser slope than one which would be drawn to a curve corrected by the true force of fishing mortality, and hence its intercept will give a value that will be higher than the true one. Thus the intercept of this tangent with the abscissa will give a maximum estimate. Similar tangents have been drawn at the beginning of years VIII to XII and the maximum estimates for the total population at the beginning of each year which are given in table 12 were obtained graphically. The difference between any two consecutive estimates gives a maximum estimate of the total mortality for the earlier of these two years. Thus a maximum estimate for the total mortality in year VII is 1590.

TABLE 12

MAXIMUM ESTIMATES OF NATURAL MORTALITY IN THE OPEONGO LAKE TROUT POPULATION BASED ON THE YEAR CLASSES 1929 TO 1937.

Age	Mean virt. pop.	Mean max. pop.	Ratio max./virt.	Mean fish. mort.	Max. nat. mort.	Max. tot. mort.	Max. force nat. mort.	k min.	k max.
VII	1036	2850	2.75	369	1221	1590	0.63	0.015	0.044
VIII	667	1260	1.89	367	492	859	0.63	0.038	0.058
IX	300	401	1.34	179	72	251	0.28	0.056	0.064
X	121	140	1.16	72	28	100	0.18	0.037	0.053
XI	48	50	1.02	28	2	30	0.046	0.034	0.049

Since the catch is known (369) a maximum estimate of the natural mortality in year VII can be found as a difference between the total mortality and the catch. The maximum estimate of natural mortality in year VII is 1221.

If it is assumed that natural mortality and fishing mortality compete for the death of a given individual, then a minimum estimate for the force of fishing mortality, k min, can be calculated by substituting the required values in equations 4, 5 and 6, (page 66). At the same time a maximum value for r, the force of natural mortality, is obtained; the minimum estimate for r is, of course, zero. The estimates for r max and k min are also given in table 12. The two estimates of k give an upper and a lower limit to the mean fraction of the population of lake trout caught by the expenditure of 100 boat hours of fishing effort in lake Opeongo.

The values for the size of the population in table 12 also allow an approximate upper limit to be set to the spawning escapement, since the contribution of eggs by females of age VI and older greatly exceeds that of females of age V, the first year in which they mature. The maximum population in table 12 would have produced no more than 3.0×10^6 eggs even allowing a most generous estimate of 10,000 for the number of individuals of age V present. The virtual population given in table 12 would have produced 1.4×10^6 eggs in the previous spawning season.

The maximum estimate above was made as follows. The maximum population at the beginning of age VII was multiplied by the egg count corresponding to the age of this group the previous spawning season,

age VI. The populations for ages VIII to XI were treated in a similar manner. The twenty fish left after age XI were all arbitrarily assigned to age XIV for the purpose of determining the egg count. Finally, as mentioned above, the spawning strength at age V was set at 10,000 by extrapolation from the maximum populations estimated at the higher ages.

Fixing the maximum value for the mean spawning escapement at 3.0×10^6 places an upper limit on the average mortality rate in the first six years of life. This value is the average annual force of decrease which will reduce the value from 3.0×10^6, the maximum value for the eggs deposited, to 2850, the maximum population at the beginning of age VII. The value of this average force is 1.15 which represents a decrement of 65% annually. Fishing mortality is included in this estimate. When it is borne in mind that there is most likely to be the highest mortality in year 0 it is not likely that the total decrement in the later years of this estimate will equal the average figure. Hence the estimate of 10,000 for the maximum population at the beginning of age VI seems to offer a comfortable margin of safety since this value allows a total mortality of 71.5% within the year.

DISCUSSION

If the ideal of the optimum catch is to so regulate the fishery that the population being exploited is maintained at a level where full advantage is taken of all of the lower stories of the food pyramid, then in all probability any management of the catch of lake trout in lake Opeongo would lead to disappointment. As far as the picture can be built up from the catch records it is clear that Opeongo contains a population of lake trout so sparse that it probably has never, during the periods for which records have been kept, been large enough to completely exploit its food supply.

Although we have no knowledge as to what conditions were like, it may be presumed from comparative data (Fry 1939[b]), that in the years which preceded the exploitation of the fishery by anglers, recruitment was suppressed by the older members of the lake trout population themselves. In more recent years there are at least strong indications (figure 10) that angling can suppress recruitment by reducing the spawning stock. It can be judged from the standard curve of availability for the lake trout in Algonquin Park lakes (Fry and Chapman 1948) that the numbers of lake trout of fishable size in lake Opeongo was

somewhere in the neighbourhood of the maximum in 1936. There is no evidence that the growth rate has increased with the reduction of the population which has taken place since that time.

It would appear, therefore, that the approach to management of the Opeongo fishery should be to reinforce the recruitment by either increasing spawning escapement, supplementing spawning escapement, or reducing natural mortality at ages before the lake trout enter the fishery. Just what methods will be effective in bringing about improvement of recruitment is not known. At present the experimental planting of yearlings is being carried out. At the same time natural spawning is being studied and clues are being sought as to what are the most vulnerable periods in the life of the lake trout and what are the chief causes of early mortality.

Just how successful the attempt to describe the Opeongo lake trout population has been can be better judged by those who can view the results with greater objectivity than can the author. None of the data collected appear to be superfluous for the objective. Determination of the age composition of the catch appears to offer great advantage, particularly in dealing with the problem of recruitment.

In the case of the Opeongo lake trout population, analysis by the methods used here seems to be greatly favoured by the simplicity introduced by a low rate of natural mortality. Even moderate rates of natural mortality would make the spread between the maximum and minimum estimates too great to be of any practical value. The estimates of spawning escapement would also be far less influenced by the fishery than would be indicated by the estimates employed there. However it is to be hoped that when sufficient data are at hand to give assurance that the curve presented in figure 11 is stable that trial values of k below k max can be employed until the curve is found which gives mortalities from which a value of k can be derived which will be identical with the trial value chosen. Such a process requires first a formulation of the course of the curve between abundance and removal since the graphical method probably does not afford tangents which are sufficiently precise.

SUMMARY

1. This report is based on creel census records of the lake trout (*Cristivomer namaycush*) fishery of lake Opeongo together with data on the age, growth and fecundity of this species in that lake.

2. The various statistics summarized below have been computed from the data by means of the formulas given in the appendix.

3. Opeongo lake trout first enter the fishery at age III but are not taken in numbers until age V. Fifty percent of the fishery is drawn from ages VII and VIII. No trout over age XVII have been taken.

4. The total contribution which a year class makes to the fishery has been termed the virtual population of that year class. The virtual population in the lake at any one time is all those fish alive at that time which are destined to be captured. It is determined by summing up the contributions to the fishery from the various year classes.

5. When the catch for a given year and the size of the virtual population are known, maximum values can be calculated for the level of exploitation (percentage removed in a given year) and for the force of fishing mortality (fraction removed per unit effort).

6. The maximum value of the force of fishing mortality varies with the age of the fish. It is very low for fish of ages III and IV, reaches a maximum for age IX and probably decreases somewhat at higher ages.

7. When mean values are known for the force of fishing mortality and for the amount of fishing effort, estimates can be made of the probable fraction removed from virtual population up to the end of the current season. When this fraction reaches one half, the estimate appears stable enough to allow prediction of the total yield from the year class.

8. The yield per unit effort varies with the time of year. It is typically low in May, is highest in late June and falls again in August and early September. Fishing may recover briefly in late September. Fishing activity is negligible from October to May.

9. There is a close relation between catch per unit effort and the size of the virtual population in those age groups above year VII for which the data are adequate. The correlation is poor or non-existent at age VII and below. This lack of correlation is attributed to annual variations in catchability which appear to be more pronounced in the younger age groups.

10. The age-length and age-weight relation of the Opeongo lake trout taken by angling are described.

11. A minimum estimate of the production of trout flesh in lake Opeongo in the years 1936, 1937 and 1938 has been made from the growth curve and the size of the virtual population.

12. Lake trout in lake Opeongo mature over years IV to VII. A certain percentage of the population of mature fish fails to develop spawn in certain years. Egg counts in relation to age are presented.

13. Two estimates of the spawning escapement are presented, one based on the eggs produced by the virtual population, the other on the size of the catch and the effort required to take it.

14. These estimates indicate that the spawning escapement dropped progressively from 1935 to 1941 recovered to the 1937 level in the years from 1942 to 1945 and again decreased sharply when the fishing intensity increased again after the war.

15. For the spawning years 1935 to 1939 a positive correlation was found between the estimated spawning escapement and the resulting year class strength.

16. A maximum estimate of natural mortality can be obtained by application of DeLury's (1947) method to the relation between catch and yield per unit effort corrected by the maximum force of fishing mortality. This is presented for ages VII to XI.

17. A maximum estimate for spawning escapement was found from the maximum estimate for natural mortality and the maximum natural mortality for the years earlier than age VII deduced.

18. It is concluded that the Opeongo lake trout fishery thoroughly exploits a sparse population and that fishing mortality far outweighs natural mortality after the population has entered the fishery. There is grave danger that the level of fishing intensity has reduced the spawning escapement to a point which affects the strength of the year classes now entering the fishery. It is therefore recommended measures be taken to supplement the spawning escapement or to reduce the natural mortality in the younger year classes.

19. These data are presented as an example of the use of commonly collected biological and fishery statistics to follow a fishery and to determine its effect on the population it exploits.

APPENDIX

This mathematical analysis of the Opeongo catch statistics has been based primarily on the die-away curve following the conventional method of analyzing such statistics which on this continent has been widely employed particularly in recent years e.g. Thompson and Bell (1934) Ricker (1940, 1944, 1948) Schaeffer (1943) and DeLury (1947). In the interests of uniformity the symbols used conform to those of DeLury who appears to have published the most general formula among the groups referred to above.

Changes in the total population of trout may arise from the following sources:

(a) An increase may come from recruitment of young fish or from immigration.

(b) A decrease results from emigration, removal by fishing, or from death by other causes which will be termed natural.

When the ages of the captured fish are known it is possible to consider year classes separately and hence if the population is confined to a discrete body of water to eliminate from consideration all changes other than a decrease due to fishing and natural mortality.

A mathematical model of the way such a decrease takes place can be simplified by making the following assumptions:

1. The rate at which fish of an age group are caught at a given instant is proportional to:
   (a) the numerical strength of the age group at this instant;
   (b) the intensity of fishing effort at this instant.
2. The rate at which fish of an age group die of natural causes is proportional to the numerical strength of the age group at this instant.
3. Such characteristics as mortality rate, egg count and response to fishing depend only on the age of the fish.

The notation used to state these assumptions is taken from DeLury 1947:

x = the age of a fish. x can, therefore, have any positive value but in actual practice it will be given integral values only e.g. 0, I, II . . .
t = the time in years, for convenience the greatest value of t used is 1 year.
T = calendar year of capture.
$N(x,t;T)$ is the number of fish aged x at time t within calendar year T.
$N(x,0;T) = N(x;T)$ is the number of fish aged x at the beginning of year T.
$e(t;T)$ = fishing intensity at time t after beginning of year T.
$E(t;T) = \int_0^t e(t;T)\,dt$ = total effort expended in year T up to time t. (the unit of effort is 100 boat hours).
$E(1;T)$ will be shortened to $E(T)$.
$C(x;T)$ = Availability, number of fish aged x captured per unit effort.
$C(T) = C(x;T)$ summed over all values of x which appear in the catches.
$r(x;T)$ = force of natural mortality affecting fish aged x in year T.
$k(x;T).e(t;T)$ = force of fishing mortality affecting fish aged x at time t within year T. $k(x;T)$ is thus the fraction of the population aged x captured per unit of effort.

$m(x;T)$ = number of eggs per individual aged x in year T.
$R(x;T)$ = number of fish aged x dying of natural causes in year T.
$K(x;T)$ = number of fish aged x caught in year T.
$D(x;T) = R(x;T) + K(x,T)$ = total decrease of fish aged x in year T.
$V(x;T) = K(x;T) + K(x + I; t + I) + K(x + 2; T + 2) + \ldots$ to the end of the table. (1)

$$\frac{K(x;T) \times 100}{V(x;T)} = \text{maximum percentage level of exploitation.}$$

Assumptions 1 and 2 imply that

$$\frac{dN}{dt}(x,t;T) = -[r(x;T) + k(x;T)e(t;T)]N(x,t;T) \qquad (2)$$

which integrates to give

$$N(x,t;T) = N(x;T) \exp\{-r(x;T)t - k(x;T)\cdot E(t;T)\} \qquad (3)$$

it follows that

$$D(x;T) = N(x;T)[I - \exp\{-r(x;T - k(x;T)\cdot E(T)]\} \qquad (4)$$

If in addition, the force of fishing mortality bears to the force of natural mortality a reasonably constant ratio,

$$R(x;T) = \frac{r(x)}{r(x) + k(x)\cdot E(T)} \cdot D(x;T) \qquad (5)$$

$$K(x;T) = \frac{k(x)\, E(T)}{r(x) + k(x)\cdot E(T)} \cdot D(x;T) \qquad (6)$$

A special case arises when $r(x) = 0$ for all values of x. Then

$$R(x;T) = 0 \text{ for all } x \text{ and } T \qquad (7)$$

which means that

$$V(x;T) = N(x;T) \text{ and} \qquad (8)$$

$$N(x,t;T) = N(x;T)[\exp\{-k(x;T)E(t;T)]\} \qquad (9)$$

then

$$K(x,T) = N(x;T)[1 - \exp\{-k(x;T)E(T)]\} \qquad (10)$$

and

$$1 - \frac{K(x;T)}{V(x;T)} = \exp\{-k(x;T)E(T)\} \qquad (11)$$

The spawning escapement ratio $S(u,v)$ between two years $T + u$ and $T + v$ will be defined by

$$S(u,v) = \frac{\sum m(x;T + u)N(x;T + u) \exp\{-k(x)E(T + u)\}}{\sum m(x;T + v)N(x;T + v) \exp\{-k(x)E(T + v)\}} \qquad (12)$$

where the summation is over all values of x. If $k(x) = k$ and $m(x;T + u) = m(x)$ for all u then we have approximately

$$\frac{m(x)K(x;T + v)}{m(x)k(x;T + u)} \div \frac{m(x)N(x;T + v)[1 - \exp\{-kE(t + v)]\}}{m(x)N(x;T + u)[1 - \exp\{-kE(t + u)]\}} \qquad (13)$$

(12) then becomes

$$S(u,v) = \frac{\sum m(x)K(x;T + v)[\exp \{-kE(T + u)\} - 1]}{\sum m(x)K(x;T + u)[\exp \{-kE(T + v)\} - 1]} \tag{14}$$

REFERENCES CITED

Applegate, V. C. 1947. Growth of some lake trout *Cristivomer n. namaycush* of known age in inland Michigan lakes. *Copeia.* 237–241.

DeLury, D. B. 1947. On the estimation of biological populations. *Biometrics 3:* 145–167.

Fry, F. E. J. 1939a. The position of fish and other higher animals in the economy of lakes. *Problems of lake biology* (Pub. 10 A. A. A. S.): 132–142.

Fry, F. E. J. 1939b. A comparative study of lake trout fisheries in Algonquin Park, Ontario. *Univ. Toronto Studies Biol. Ser. 46, Pub. Ont. Fish. Res. Lab. 58:* 69 pp.

Fry, F. E. J. and V. B. Chapman. 1948. The lake trout fishery in Algonquin Park from 1936 to 1945. *Trans. Am. Fish. Soc. 75:* 19–35.

Fry, F. E. J. and W. A. Kennedy. 1937. Report on the 1936 lake trout investigation, lake Opeongo, Ontario. *Univ. Toronto Stud. 42, Pub. Ont. Fish. Res. Lab. 54:* 20 pp.

Juday, C. 1930. Growth studies of game fish in Wisconsin waters. Notes from the Biological Laboratory of the Wisconsin Geological and Natural History Survey. March 1930. Mimeographed.

Kennedy, W. A. 1943. The whitefish, *Coregonus clupeaformis* of lake Opeongo, Algonquin Park, Ontario. *Univ. Toronto Studies Biol. Ser. 51, Pub. Ont. Fish. Res. Lab. 62:* 43 pp.

Miller, R. B. and W. A. Kennedy. 1948. Observations on the lake trout of Great Bear Lake. *J. Fish. Res. Bd. Can. 7:* 176–189.

Ricker, W. E. 1940. Relation of "Catch per unit effort" to abundance and rate of exploitation. *Fish. Res. Bd. Can. 5:* 43–70.

Ricker, W. E. 1944. Further notes on fishing mortality and effort. *Copeia:* 23–44.

Ricker, W. E. 1948. Methods of estimating vital statistics of fish populations. *Indiana Univ. Publ. Sci. Ser. 15:* 101 pp.

Royce, W. E. M. S. The reproduction and studies of the life history of the lake trout *Cristivomer namaycush* (Walbaum). A thesis presented to the Faculty of the Graduate School of Cornell University for the degree of Doctor of Philosophy. 1943.

Schaeffer, M. B. 1943. The theoretical relationship between fishing effort and mortality. *Copeia* 79–82.

Surber, T. 1933. Rearing lake trout to maturity. *Trans. Am. Fish. Soc. 63:* 64–68.

Thompson, W. F. and F. H. Bell. 1934. Biological statistics of the Pacific halibut fishery. *Rpt. Int'l. Fish. Comm. 8:* 47 pp.

Various Authors. 1947. North West Canadian Fisheries surveys in 1944–1945. *Bull. 72 Fish. Res. Bd. Canada:* 94 pp.

QUERIES

QUERY: I read the paper by Anderson and Manning in the
64 September 1948 *Biometrics* and tried a simplified method of analysis as follows:

(1) At each location take the sum of the yields of the early planting minus the sum of the yields of the late planting. Consider this difference, D, to be located at the date of the intermediate planting.

(2) Plot the results, and observe that a linear regression seems adequate. Fit a linear regression, with the result

$$D = 4.04 + 0.72(t - \bar{t}), \qquad s^2 = 1.89 \text{ on } 6 \text{ d.f.}$$

Hence the coefficients are 4.04 ± 0.49 and 0.72 ± 0.22.

If Anderson and Manning's formula for regression of yield on time is accepted, then we have

$$D = y_{t(i-1)} - y_{t(i+1)} = 8b + 16c(t_i - \bar{t}) = 4.04 + 0.76(t - \bar{t}).$$

Thus the simple method yields the same estimate of the first coefficient and a nearby estimate of the second coefficient. The ratio of coefficients to their estimated standard errors is 8.24 and 3.27 for the simple method as against 6.73 and 2.92 for the more complex method.

I should like to know the answers to the following questions:

(1) Am I right in supposing that the simple method has proved more accurate in this case?

(2) How would I expect the accuracies to compare in general?

(3) Is the following picture correct?— The three plantings at one locality provide: a mean yield which is wholly confounded with locality; an average slope, which depends *only* on the early and late plantings, and which is confounded with the linear effect of *Lygus*; a quadratic effect, which produces an estimate of the quadratic regression coefficient of low precision. These three results may be analyzed separately. Thus the best estimate of the quadratic regression coefficient is a weighted combination of this latter estimate, and the estimate, already found above, which is based on the slope of the "observed slope" vs. "mean time of planting" plot. (The quadratic effects in each location give 0.205 ± 0.101 for the coefficient, as compared with 0.0450 ± 0.0138 from the plot. The weighted mean is 0.0479 ± 0.0135).

(4) Why was it worthwhile to make the complicated analysis?

ANSWER: I should indicate, before answering the specific questions, that it can be shown that the estimate of b is exactly the same using equation (20) in the Biometrics article and using that of the querist. This can be proven by expressing the value of b given in Table 7 in terms of the locality-date totals of Table 1. Similarly the equation for c given in Table 7 is almost the same as that given by the querist. In fact from Table 7

$$\frac{4096c}{3} = \frac{2}{3}[(11y_{11} - y_{21} - 10y_{31}) + (8y_{22} - y_{32} - 7y_{42})$$

$$+ (5y_{33} - y_{43} - 4y_{53}) + \cdots].$$

As compared to querist's,

$$1344c = [7(y_{11} - y_{31}) + 5(y_{22} - y_{42}) + 3(y_{33} - y_{53}) + \cdots].$$

Now let me answer the questions in the order given by the querist.

(1) The simple method cannot possibly be more accurate, since it never utilizes more information than does our equation (20). The accuracy of the estimate of b will be exactly the same in the two cases, while that of c will be slightly greater using our equation. The reason for querist's apparently greater accuracy is that he has used a different estimate of σ^2 than that used in the article. I could have used the deviation from regression mean square [equation (28) in the article] as the error term, but since it was smaller than the experimental error I decided that the experimental error was a better estimate of the error variance for both b and c. The reason for this was that the deviation from regression mean square should equal the experimental error plus any extra component due to the regression. Hence when this deviation mean square was smaller than the experimental error, I concluded that we should use the experimental error, which was estimated with 48 degrees of freedom instead of the 7 degrees of freedom for the deviation [see equation (28) in the article]. It should be noted that querist's deviation mean square (0.236 on a per plot basis) is less than mine. It seems unreasonable to use this very low estimate, which neglects the failure of his prediction equation to fit the yields for the middle planting date at each location.

(2) As stated above the variance of b in both cases is $\sigma^2/64$. The variance of c using our equation (20) is $3\sigma^2/4096$, and using

querist's equation $\sigma^2/1344$. Hence the efficiency of the latter is $4032/4096 = 0.985$. This shows that the loss of information using the simpler equations is only 1.5%.

(3) The value of c given by our equation (20) is a weighted average of the two estimates mentioned by the querist. I might mention at this stage that there seems to be a slight error in the value of C given in Table 7. Apparently $C = 2299.82$ instead of 2299.62. Hence c equals -0.04722 instead of -0.04737.

(4) There is little to recommend one over the other of the two analyses if the regression is truly quadratic. However, a loss of information in neglecting the middle planting dates becomes more important if a higher degree equation is required. A preliminary examination using our equation (38) shows the relative efficiencies for b, c, d, and e are about .96, .84, .92 and .80 respectively. Also I doubt if there is actually much more work in computing the estimates of the constants in our equations once the solutions given at the bottom of Table 7 or the inverse matrices (40) and (43) are known. Hence if future experiments were run, these solutions could be used without going through the preliminary matrix inversions. And even though the middle dates are not used for estimation purposes, they should be used in determining the goodness-of-fit.

Finally, I should mention that one of the main purposes of writing this article was to obtain suggestions for an improved design in which the range of planting dates is no more than four weeks at a given location. The experiments analyzed here were set up before an analysis had been devised, but we hope that something better can be offered for the future. As indicated on pages 194–195, I tried several other 3-date designs but could find none to be superior to the one which had been used. It is here that the querist missed the most important part of his idea, namely that we use our same design but omit the middle date at each location. The extra plots could be used to make six replications at each location, thus increasing the efficiency of the experiment by almost 50% if the quadratic equation were adequate

$$\left[\sigma^2(b) = \frac{\sigma^2}{96}, \qquad \sigma^2(c) = \frac{\sigma^2}{2016}\right].$$

If it were possible to find four more locations so that four replications were used at each of 12 locations, $\sigma^2(b)$ is the same but $\sigma^2(c) = \sigma^2/4576$. Since this involves a range of 26 weeks in planting dates, it is doubtfu

if a wider range of planting dates should be used. However, if it were possible to separate the two plantings at each location by six weeks, using 8 locations and 11 planting dates, we could reduce $\sigma^2(b)$ to $\sigma^2/216$ and $\sigma^2(c)$ to $\sigma^2/4536$.

R. L. ANDERSON

65 QUERY: This laboratory has just completed an experiment comparing the effect of three levels of dilution of bull semen upon breeding efficiency, as measured by non-returns to first and second services within 60 to 90 days after service. In the experiment the data from one bull comprised a Latin square. Three collections (each made on a given day and consisting of the semen from two or more ejaculates combined) were made from each bull and each collection

TABLE 1

SPERM PER CC. (100,000) AND NON-RETURN PERCENTAGES

		Dilution, 1:100			Dilution, 1:150			Dilution, 1:200		
Bull	Collection	Group	Sperm	Non-return	Group	Sperm	Non-return	Group	Sperm	Non-return
1	1	2	74	57	1	50	63	3	37	46
	2	1	106	49	3	71	64	2	53	59
	3	3	125	61	2	83	67	1	63	61
2	1	3	66	66	2	75	60	1	57	65
	2	2	60	60	1	71	76	3	53	49
	3	1	75	75	3	104	67	2	78	69
3	1	1	69	69	3	107	72	2	80	78
	2	3	76	76	2	105	70	1	79	60
	3	2	72	72	1	100	63	3	75	71
4	1	1	66	66	2	66	63	3	49	62
	2	2	67	67	3	119	68	1	90	67
	3	3	74	74	1	66	77	2	49	72
5	1	3	65	65	1	85	69	2	64	62
	2	1	68	68	2	83	60	3	62	63
	3	2	74	74	3	95	76	1	71	75
6	1	2	122	71	3	81	71	1	61	66
	2	3	119	62	1	79	70	2	60	70
	3	1	140	78	2	93	69	3	70	65

was divided into three parts, with one part being diluted 1:100, one part 1:150, and one 1:200. Each third was shipped out to one third of the inseminators. The inseminators were divided into three groups as equal as possible in respect to breeding efficiency and cows bred. At time of collection a count of spermatozoan numbers was made of an aliquot undiluted sample from each collection. The numbers of sperm in the diluted samples were determined by calculation. Semen from six bulls has been used in the experiment.

I have attempted to analyze the data by analysis of variance and covariance. The table for covariance has been calculated using logarithms of sperm numbers, for the variance of actual sperm numbers was proportional to the means.

Upon examination of Table 2 you will please note that the error for Sx^2 (logs of sperm per cc.) is a negative number. The error term for Y also appears to be excessively small. Am I attempting to remove some sources of variance which are not justifiable? I would suspect that I should not be attempting to remove variance by means of all of the interactions listed in the summary tables.

The two items of information in which I am especially interested are dilution differences and bull × dilution interaction. What should be used as error term for testing their mean squares?

TABLE 2

COVARIANCE—LOGARITHMS OF SPERM/C.C. (X) AND NON-RETURNS (Y).

Source	D.F.	Sx^2	Sxy	Sy^2
Total	53	1.3130	23.00	2,653.20
Bulls (Squares)	5	0.1777	10.16	815.42
Collections	12	0.2936	5.99	673.78
Groups	2	0.0003	0.10	42.48
Dilutions	2	0.8370	7.01	128.70
Bulls × Dilutions	10	0.0016	−0.08	183.75
Bulls × Groups	10	0.0013	−0.11	468.63
Groups × Dilutions	4	0.0719	2.15	308.85
Error	8	−0.0704	−2.22	31.59

ANSWER: The calculational difficulty is this: Groups × Dilutions and Error are not orthogonal; hence, their sums of squares are not additive. Your assumption that they are additive gives rise to the negative remainder. The correct sum of squares for

Error is the sum of the last two in the table, the degrees of freedom being $4 + 8 = 12$. This will seem reasonable if you think of the 2 degrees of freedom for error in each of the 6 squares (bulls), 12 in the pooled sum of squares.

I don't see why you have transformed the sperm numbers to logarithms. The distribution of the independent variable is not important. On the contrary, the distribution of the dependent variable affects the distribution of F: transformation to angles is worth considering. However, unless the numbers of cows in the cells of your table are very small, the transformation will not likely change decisions because your percentages lie near the middle of the range and do not vary greatly.

Since the sperm numbers for the two higher dilutions are calculated rather than observed, their use as a covariate should be avoided. The sperm numbers for these two dilutions contain no information not already used in the first dilution so that the calculations would have to be modified. Not only so, but I suspect that the dilution numbers are measured with considerably more error than are the dilution ratios. This raises a question as to their value as a covariate—you might gain nothing by the covariance analysis even if you had made independent observations of sperm numbers in every cell of the table; unless, indeed, you had replicated the observations sufficiently to get rather accurate determinations.

In your design you did not use all combinations of bulls, collections, dilutions and groups. If all were present the number of cells in the table would be $(6)(3)(3)(3) = 108$ instead of your 54. This implies some fractional replication which complicates the analysis. A suitable linear hypothesis is this:

$$y_{ijkl} = \mu + \beta_i + \gamma_{ij} + \delta_k + (\beta\delta)_{ik} + g_l + (\beta g)_{il} + (\delta g)_{kl} + \epsilon_{ijkl}$$

where

μ = mean effect
β_i = bull effect ($i = 1 \cdots 6$)
γ_{ij} = collection effect ($j = 1 \cdots 3$)
δ_k = dilution effect ($k = 1 \cdots 3$)
g_l = group effect ($1 = 1 \cdots 3$)
$\beta\delta$ = bull $\times$ dilution interaction
βg = bull $\times$ group interaction
δg = dilution $\times$ group interaction
ϵ = experimental error

and

$$\beta \text{ is } N(0, \sigma_\beta^2)$$

$$\gamma \text{ is } N(0, \sigma_\gamma^2)$$

$$\delta \text{ is } N(0, \sigma_\delta^2)$$

$$g \text{ is } N(0, \sigma_g^2)$$

$$\beta\delta \text{ is } N(0, \sigma_{\beta\delta}^2)$$

$$\beta g \text{ is } N(0, \sigma_{\beta g}^2)$$

$$\delta g \text{ is } N(0, \sigma_{\delta g}^2)$$

$$\epsilon \text{ is } N(0, \sigma^2)$$

The dilution × group interaction (with 4 degrees of freedom) may be split into two parts, each with 2 degrees of freedom, and these may be designated [following Yates' notation] by $DG(I)$ and $DG(J)$. Upon examination of the data, it becomes clear that $DG(J)$ is confounded with collections for bulls 1, 2 and 3 and that $DG(I)$ is confounded with collections for bulls 4, 5 and 6. It is necessary, therefore, to estimate $DG(I)$ from bulls 1, 2 and 3 only and $DG(J)$ from bulls 4, 5 and 6 only. The following analysis of variance results from this procedure:

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expectation of Mean Square
Bulls [B]	5	815.42	163.08	
Collections within bulls	12	673.78	56.15	
Dilutions [D]	2	128.70	64.35	$\sigma^2+3\sigma_{\beta\delta}^2+6\sigma_{\delta g}^2+18\sigma_\delta^2$
Groups [G]	2	43.48	21.24	
$DG(I)$	2	170.29	71.47	$\sigma^2 + 3\sigma_{\delta g}^2$
$DG(J)$	2	115.59	(average)	
BD	10	183.75	18.38	$\sigma^2 + 3\sigma_{\beta\delta}^2$
BG	10	468.63	46.86	$\sigma^2 + 3\sigma_{\beta g}^2$
Remainder	8	54.56	6.82	σ^2
Total	53	2653.20		

It should be noted that the expectation of the DG mean square is

$\sigma^2 + 3\sigma_{\delta_g}^2$ and not $\sigma^2 + 6\sigma_{\delta_g}^2$ because each of $DG(I)$ & $DG(J)$ is confounded with collections for one-half of the bulls.

The two tests which querist asks about in this particular problem are those for (i) dilutions and (ii) bulls $\times$ dilutions. The latter is easy to perform being simply:

$$F = \frac{\hat{\sigma}^2 + 3\hat{\sigma}_{\beta\delta}^2}{\hat{\sigma}^2} = 2.69 \qquad \text{with d.f.} \qquad n_1 = 10, n_2 = 8,$$

which is not significant at the .05 level. The former, however, is a composite test and an approximate (though biased) test would be as follows:

$$F = \frac{\hat{\sigma}^2 + 3\hat{\sigma}_{\beta\delta}^2 + 6\hat{\sigma}_{\delta_g}^2 + 18\hat{\sigma}_\delta^2}{\hat{\sigma}^2 + 3\hat{\sigma}_{\beta\delta}^2 + 6\hat{\sigma}_{\delta_g}^2} = \frac{64.35}{147.68}$$

$$= .44 \qquad \text{with d.f.} \qquad n_1 = 2, \hat{n}_2 = 2.12,$$

where the denominator is obtained by taking the mean square for bulls $\times$ dilutions interaction and adding to it a multiple of an estimate of the $\sigma_{\delta_g}^2$ component, this estimate being obtained in the usual way:

$$\hat{\sigma}_{\delta_g}^2 = \frac{71.47 - 6.82}{3} = 21.55$$

The estimate of n_2; i.e., $\hat{n}_2 \cong 2.12$ was calculated using the formula provided by Satterthwaite in this journal, Vol. 2, pages 110-114, December, 1946:

$$\hat{n}_2 = \frac{\{\hat{\sigma}^2 + 3\hat{\sigma}_{\beta\delta}^2 + 2(\hat{\sigma}^2 + 3\hat{\sigma}_{\delta_g}^2 - \hat{\sigma}^2)\}^2}{\dfrac{\{\hat{\sigma}^2 + 3\hat{\sigma}_{\beta\delta}^2\}^2}{k_1} + \dfrac{\{2(\hat{\sigma}^2 + 3\hat{\sigma}_{\delta_g}^2)\}^2}{k_2} + \dfrac{\{-2\hat{\sigma}^2\}^2}{k_3}}$$

$$= \frac{(147.68)^2}{\dfrac{(18.38)^2}{10} + \dfrac{(2(71.47))^2}{2} + \dfrac{(-2(6.82))^2}{8}}$$

$$= 2.12$$

The value of F obtained is not significant.

It might be noted that the mean square for the bulls $\times$ groups interaction leads to a significant value of F, but there does not appear to be any reason to expect such a result in this type of experiment.

Bernard Ostle

ABSTRACTS

59 BALDWIN, ALFRED L. (Fels Research Institute for the Study of Human Development, Antioch College). **Statistical Problems in the Treatment of Case Histories.**

In the use of case history material for research in human development, there frequently arises the need to apply statistical tests for the verification of interpretations and also for the discovery of relationships which demand interpretation. One of the problems raised by such attempts is the handling of the time dimension. Statistical problems in the treatment of case histories will, in this report, be illustrated in two areas.

1. Repeated measurements of a battery of variables on a single individual have been treated by various workers as a sample of measurements from the hypothetical "population" of measurements on that individual. In the analysis of such data the existence of temporal trends may be demonstrated. In economics such trends are removed to avoid "spurious" correlations. The handling of such data in psychological research requires a careful analysis of the assumptions involved.

2. Approximate temporal simultaneity of discrete events—for example anxiety attacks and threats of loss of status—are commonly used by the clinician as evidence for supporting an interpretation which relates those two events. The problem of selecting an appropriate statistic and statistical model will be discussed.

60 KUBIS, JOSEPH F. (Fordham University). **Statistical and Experimental Factors in the Diagnosis of Consciously Suppressed Affective Experience.**

This is a brief presentation of the development of criteria and their statistical evaluation in the diagnosis of criminal guilt. Unlike many traditional clinical problems in which the discovery of subconsciously repressed conflicts plays the major role, this practical problem concerns itself with the detection of consciously suppressed knowledge. The situation is known but the unique person to fit the situation has to be determined.

The objectification and verification of clinical intuitions in their necessary relations to both experimental and statistical control will be discussed. At present the variation in precision of clinical diagnosis is more a function of deficient experimental control and a lack of objectification of intuitions than of inadequate statistical procedures, or the unavailability of appropriate statistical techniques.

61 KOGAN, LEONARD S. and J. MCVICKER HUNT. (Institute of Welfare Research, Community Service Society of New York). **Reliability of Judges' Ratings of Case History Material.**

Using an "anchored" scale of movement, various groups have rated the movement in 38 case summaries supplied by the Family Service Department. Two major null hypotheses present themselves in studying the reliability of inter-judge ratings within and between groups: (1) do groups with varying casework experience, but trained similarly, exhibit no differences in inter-judge agreement and (2) does a single group display no difference in inter-judge agreement as a result of training?

Several approaches to the statistical solution for testing these null hypotheses have been considered:

(1) Obtain a complete matrix of inter-judge correlations within each group, treat these correlations as scores, and apply t-tests.
(2) Treat the standard deviations of each set of ratings for a given case as scores and apply t-tests to the arrays of the two groups.
(3) Apply analysis of variance to each group and compare the several variance estimates from one sample with that of the other.

These and similar techniques are all questionable procedures because of the assumptions involved and more appropriate methods would be valuable.

62 RABIN, ALBERT I. (Michael Reese Hospital, Chicago). **Statistical Problems Involved In Rorschach Patterning.**

The present discussion is an outgrowth of efforts made to investigate the objective Rorschach criteria for schizophrenia. Hitherto the diagnosis of schizophrenia (and other disorders) by means of the Rorschach test has been made on the basis of—

(a) A few factors in which the disorder differs from normalcy and other conditions, with statistical significance.
(b) The presence of certain "configurations" of those and other variables which only the "experienced Rorschacher" can detect.

In the first instance, there is an overlapping with other disorders; in the second instance, the configurations detected follow, if not a strictly intuitive, certainly an insufficiently objective or quantitative method.

In order to reduce the second method to a more scientific procedure, a cluster analysis has been undertaken. The difficulties encountered in this method will be discussed; especially—

(a) The insufficiency of present clusters for differentiation.
(b) Quantification of qualitative (content) data.
(c) The importance of the "single" or of the "startling" symptom or "sign".

63 CRONBACH, LEE J. (Bureau of Research and Service, College of Education, University of Illinois). **Statistical Problems in Multi-Score Tests.**

An increasing number of clinical tests report the subject's performance in a set of scores. If n people are given k scores, the resulting data can be plotted as n points in k-space. Statistical methods are necessary to describe the features of such a distribution, to compare two distributions from different groups, and to relate the distribution of cases in one k-space (test) with their distribution in another k'-space (criterion). There are serious limitations to the methods which have been used in the past to simplify this problem. These methods include treating the scores one at a time, the signs approach for limited patterns of scores, the Vernon matching method, and the discriminant function. Characteristics of a desirable statistical system for processing current clinical data are considered. Two new procedures are described: a more complex matching technique, and a pattern tabulation technique for dealing with sets of three normalized scores simultaneously.

64 STEPHENSON, WILLIAM. (Visiting Professor of Psychology, University of Chicago). **A Statistical Approach to Typology.**

Trait-universes are first defined. Thus, a limited trait universe of 2000 traits can be listed for Jung typology; 500 terms current in Rorschach interpretations likewise comprise a trait-universe, as can 800

everyday terms which one might use instead of the Rorschachian technical language. Samples from such universes can be quantified, with a person's behavior, or his personality structure, as the variables. These, when correlated with others, provide the basis of systematic typology. It is easy in this way to represent in one correlational table all the cases described, for example, in Beck's well-known monograph "*An Introduction to the Rorschach Test*". Types appear as common (non-fractional) factors, which may, or may not, be correlated factors.

Typology is thus presented as systems of trait-universes, with those concerned with motivation on the core, so to speak, and others concerned with immediate behavior at the periphery, and with many others in between. Types can appear at all levels in the system, within definable limits of error. The statistical matters involved are examined. The concept of 'significance', used for these quantifications, is defined as having the same mean value for all personalities.

The system can adequately describe all the major theories concerning personality types; it allows unique personalities to be represented; and provides a basis for the study of the intuitions of clinical and other psychologists.

65 HORN, DANIEL. (American Cancer Society). **Intra-Individual Variation in the Study of Personality.**

The psychometric approach to the understanding of an individual is limited, not by the mathematical statistical tools available, but rather through the failure to collect data for analysis in a fashion that matches a dynamic theory of personality. Dynamics implies change and the study of change necessitates a series of measurements taken over a period of time. A dynamic relationship can be evaluated statistically only by serial observations of the behavioral acts under study. The collection of data for the study of intra-individual variation and co-variation offers a way of applying present statistical techniques to current dynamic theories of personality. Otherwise a statistical analysis of cross-sectional or inter-individual data tacitly assumes a universality of static personality traits on a level with faculty psychology of the 19th century.

66 GUETZKOW, HAROLD. (University of Michigan). **Unitizing and Categorizing Problems in Coding Qualitative Data.**

The transformation of qualitative data obtained in interviews, autobiographies, free-answer questions, projective materials, and typescripts

of group meetings into a form which renders them susceptible to quantitative treatment constitutes "coding". Coding procedures implicitly involve two operations, that of separating the qualitative material into codable units, and of establishing systems of categories which can be applied to the unitized material. Examination of particular procedures suggests generalizations about the construction of category systems and the use of unitizing operations. In addition to these generalizations, it is possible to derive reliability estimates. These estimates also aid the investigator in deciding how large a sample of his data needs to be check-coded to insure the desired level of accuracy.

67 BROWN, GEORGE W. (The Rand Corporation). **Basic Principles for Construction and Application of Discriminators.**

Let two populations A and B have specified multivariate distributions given by $f_A(x, y, z, \cdots)$ and $f_B(x, y, z, \cdots)$, respectively. To classify an individual into A or B on the basis of his observed $(x, y, z, \cdots)$, form the likelihood ratio $\lambda = f_B(x, y, z, \cdots)/f_A(x, y, z, \cdots)$, choosing B if $\lambda > \lambda_0$, A if $\lambda \leq \lambda_0$. This process is optimum in the sense that no other procedure can have a smaller probability of misclassifying an "A" as a "B" without having a larger probability of misclassifying a "B" as an "A", and conversely. When f_A and f_B are multinormal, with the same variances and covariances, this process leads to a discriminator which is equivalent to the usual discriminant function.

The above process is optimum in another sense, since (if λ is properly chosen) it minimizes the expected risk, when costs of misclassification and proportions of A's and B's in the population to be classified can be estimated.

If C_A is the cost of misclassifying an A, C_B is the cost of misclassifying a B, and p is the expected proportion of A's, then λ should be taken as $(pC_A)/((1 - p)C_B)$. Thus the choice of λ can, in some cases, be made on a rational basis. Examples given.

The procedure is extended to include the classification of an individual into one of several populations.

68 GARDNER, ERIC F. (Syracuse University). **Some Comments on Scaling Theory with a Proposal for a New Type of Scale.**

Numerous instruments have been made available to the clinician and guidance counselor to aid him in his attempt to determine the

interests, attitudes, aptitudes and achievement of his clients. To interpret the results obtained with these instruments in terms of change within and differences between individuals, scales are needed.

There have been two general approaches to the scaling problem. A direct approach, in which the scale maker constructs his instrument in such a way that the score yields a scale directly without conversion, has been used in general by makers of attitude scales. Such scales generally furnish information concerning the relative rank order among the individuals to whom it is administered. Various techniques to determine scalability have been devised. A brief review of the methods of such workers as Guttman, Likert, and Thurstone will be given.

The second or indirect approach has been used by the makers of aptitude and achievement tests. In order to measure growth, interval scales are desired. The relationship between the units used and the shape of the frequency distribution of a particular trait has been used extensively in the past to define specific trait units. The work of Thorndike, McCall, and Flanagan, in which normality of distribution has been hypothesized will be discussed.

A method for the determination of a new type of scale unit which does not necessitate the assumption of a normally distributed trait will be described. (This exposition will constitute the major portion of the paper). The value of this type of scale to the clinician and counselor will be discussed.

69 VOTAW, D. F., JR. (Yale University). **Compound Symmetry Tests in the Multivariate Analysis of Medical Experiments.**

If experimental quantities (e.g., % CO_2 in blood, hematocrit, etc.) are measured several times on each member of a sample (of animals, say), the experimenter may wish to test statistically whether these quantities are "stable with respect to time". It is assumed that the sample is drawn from a population having a normal multivariate distribution. The (null) hypothesis of "stability" can be interpreted as a hypothesis of "compound symmetry" in the distribution. The distribution theory of criteria for testing compound symmetry was given in a recent paper [see Ann. Math. Stat., Vol. 19, pp. 447–473]. The present paper gives simple methods of computing the criteria and gives the exact cumulative distribution functions for several of the criteria (when the corresponding null hypotheses are true).

70 COLLINS, SELWYN D. (Public Health Service). **Intensive and Extensive Morbidity Surveys.**

Morbidity data for the whole country, or for a representative sample of it, certainly would be more useful than local surveys. But the techniques of collecting morbidity data in intensive local surveys are hard to apply simultaneously in the many localities needed for a good sample.

Sickness ranges from minor nondisabling colds to chronic incapacitation with confinement to bed or to a hospital. Because of the difficulty of defining what is to be recorded as a case, rates obtained in different surveys vary considerably.

Nondisabling cases in the United States have considerable medical care. However, data on such acute cases are not so necessary, but important chronic diseases do not keep the patient from work until the later stages. Most chronic diseases are curable only in the early stages, so chronic nondisabling cases are important. A correct diagnosis and the length of time since first contracted, as well as the duration of disability, are important in recording chronic diseases.

It is essential that the transition from intensive local surveys to extensive national surveys be not accompanied by a neglect of important kinds of data.

71 SANDERS, BARKEV S. (Federal Security Agency). **Measurement of the "Memory" Factor in Morbidity Surveys.**

Incidence of illness for a year was derived from illnesses of specified duration current on the day of enumeration by first differencing. Completed illnesses reported in the survey as compared with the estimate was grossly deficient. Only 15 per cent of illnesses disabling 7-13 days had been reported; 43 percent of illnesses lasting 30-59 days and 75 percent of those lasting 91 to 181 days. The reported illnesses of all durations represented 26 percent of the estimated total, prior to correction for seasonality. When correction was made for seasonality, the completed illnesses reported over the year represented 59 per cent of the total. The underreporting is so great even for illnesses causing disability for 2-3 months that it does not seem that it can be attributed solely to forgetfulness. All future surveys should avoid obtaining information on disabling conditions over a year's period. They should concentrate on obtaining information on disabling illness prevalent on the day of the visit, and how long these disabilities have lasted.

72 C. I. BLISS and NEELY TURNER (Connecticut Agricultural Experiment Station) and D. F. VOTAW (Yale University). **Dosage-Mortality Curves from Counts of Survivors.**

The calculation of dosage-mortality curves is described for experiments in which only the survivors can be counted at each dosage level and the number exposed to each dose must be inferred initially from the number of survivors on untreated checks. The problem has been solved with the following assumptions. (1) In the absence of treatment, the variation in the number of larvae per plot is Poisson with m = the expected number per plot. (2) On the plots receiving any given dose the number of larvae is also subject to Poisson variation with mq = the expected number, where q is the proportion expected to survive. (3) The proportion of survivors is a function of the log-dose involving the normal distribution, so that the expected probit is $Y = \alpha + \beta X$, where X is the log-dose of toxicant and Y is the deviate (+5) corresponding to the proportionate area q of the normal curve.

Maximum likelihood has been used to obtain estimates of m, α and β, beginning with m_0 = the number of survivors on the untreated check and a graphic estimate $Y = a_0 + b_0X$. Following an initial estimate (m_1) of m, the problem is reduced to the solution of two simultaneous equations to find corrections which are added to a_0 and to b_0 to obtain a_1 and b_1. The procedure can be carried to i approximations ($i = 0, 1, 2, \cdots$) and leads to a χ^2 test of the agreement between expected and observed results and to estimated variances of a_i and b_i. The calculation is illustrated with data from a dusting experiment on the Mexican bean beetle.

73 GREEN, MELVIN W. (American Pharmaceutical Association Laboratory, Washington, D. C.) and LILA F. KNUDSEN. (Food and Drug Administration, Federal Security Agency, Washington, D. C.). **Statistical Variations in Contents of Dry-Filled Ampuls in Current Pharmaceutical Practice.**

In 1944, the Canadian government amended the Food and Drug Act to provide weight tolerances for the contents of dry-filled ampuls. In 1946 one of our Federal agencies promulgated similar standards for a limited number of medicinal agents. In neither case was the size of the sample clearly defined, nor was it clear whether the proposed tolerances should be based upon averages or individual ampuls, although the latter was strongly implied.

To test current production against these standards and to provide a more rational basis for such standards, 612 manufacturers' lots from 5 different firms and representing 7 drugs at different dosage levels were examined. Samples of 10 were taken from each lot. About half of the lots examined met the proposed standards for individuals.

The data were subjected to variance analysis and a study of the operating characteristics. When the standard deviation (corrected to include both between-lot and within-lot variation) was plotted against the labeled quantity, it was observed that this standard deviation was substantially constant beyond about 150 mg. This suggests limits in terms of per cent below 150 mg. and in terms of a definite and constant weight above 150 mg.

For illustrative purposes, three-sigma limits were then established in one case based upon this standard deviation. Operating characteristic curves were drawn by plotting the probability of acceptance against deviation from a predetermined limit. These curves were drawn so that P_a = 50 per cent at the limit. Examples of several such curves were shown.

74 IPSEN, J. (Yale University). **A Practical Method of Estimating the Mean and Standard Deviation of Truncated Normal Distributions.** (To appear in Human Biology).

Biological data often present themselves as incomplete or truncated distributions. In the total sample, a certain number of individuals have been measured, but the remainder are known only to be all larger (or all smaller) than a known value which is the inherent truncation point. A method of estimating the mean and the standard deviation from such data is given, and tables to aid the calculation are provided.

75 TUKEY, JOHN W. (Princeton University). **Separating Means into Two Groups.** (Submitted to Biometrics as "Comparing individual means in the analysis of variance.")

A method of determining the significance of differences between adjacent means in a group, and of testing the significance of deviation of a straggling mean from the others of a group, is given. The test of significance is approximate. There is a worked example.

76 TUKEY, JOHN W. (Princeton University). **Testing Row-by-Column Tables for Non-Additivity.** (Submitted to Biometrics as "One degree of freedom for non-additivity.")

A scheme of computation is given for picking out a single degree of freedom from row-by-column tables which corresponds to non-additivity. There is a worked example.

77 CORNFIELD, JEROME and NATHAN MANTEL. (Public Health Service). **Simplified Methods for Computing the Maximum Likelihood Estimate of the Dosage-Response Curve.**

The equations defining the maximum likelihood estimates of the ED_{50} and slope of the probit line can be solved only by an iterative process. A process due to Garwood is known to converge to the correct solution with fewer cycles of computation than the better known Bliss-Fisher solution. In its original form Garwood's solution involved more computation per cycle. This paper presents tables, covering the range 0(.01)10 probits, which reduce the amount of computation per cycle so that it is now possible to take advantage of the more rapid convergence of Garwood's solution.

In many cases an extension of Karber's method, which can be derived from the maximum likelihood equations by means of three simplifying assumptions, can be used to obtain provisional estimates of the constants of the probit line. In experiments involving small numbers of animals this procedure appears to provide more accurate initial approximations than the usual graphical methods. In situations in which maximum likelihood solutions are not required, it also appears that this extension can often provide acceptable estimates with a considerable reduction in computation.

A study of Karber's method extended indicates that for finite samples the method of maximun likelihood yields biased estimates of the slope of probit line. An alternative estimation procedure which has been proposed, minimum chi-square, is subject to even more serious biases. Consequently, a modification of the method of maximum likelihood to eliminate the bias in the slope estimate is suggested.

78 HARSHBARGER, BOYD. (Virginia Polytechnic Institute). **Standard Errors Applicable to Rectangular Lattice Designs and Triple Rectangular Lattice Designs.**

In the Rectangular Lattice Designs and the Triple Rectangular Lattice Designs there are more than two equations for the variances of the difference between varietal means.

This paper presents the equation for the variances of the differences

between varietal means for all possible cases and gives a simple method for assigning the right combination of the variety means to the proper formula.

In the Rectangular Lattice Designs there are, in general, four equations. In the Triple Rectangular Lattice Design there are, in general seven equations.

79 BERKSON, JOSEPH, M. D. (Division of Biometry and Medical Statistics, Mayo Clinic, Rochester, Minnesota). **Minimum X^2 and Maximum Liklihood Solution in Terms of a Linear Transform, with Particular Reference to Bio-Assay** (Abstract of paper submitted to the Journal of the American Statistical Association).

A solution is derived for estimating the parameters of any function $q_i = F(x_i, \alpha, \beta)$ in terms of a linear transform $Y_i = \alpha + \beta x_i$, fulfilling the criterion of maximum X^2, similar to the maximum likelihood solution of the integrated normal curve in terms of probits previously given by Bliss and Fisher. The solution involves successive iteration with use of specified weights and working values just as does the maximum likelihood probit solution. A table is presented giving the weight and working value for the minimum X^2 solution and the maximum likelihood solution for a number of functions in common statistical use.

80 BERNSTEIN, MARIANNE E. (Purdue University). **Recent Changes in the Secondary Sex Ratio in the Upper Social Strata.** (In print Human Biology).

On a stratified sample of 3898 families belonging to the American and German upper social strata, statistical investigations were made as to the effect of birth order, family size, and improved living conditions on the secondary sex ratio of Man. The data represent random samples from "Who is Who in Commerce and Industry", and "Wer Ist's", and a statistical technique was devised to check their reliability. The expected number of 1- to 4-child families with sons only is

$$S = N_1p^1 + N_2p^2 + N_3p^3 + N_4p^4$$

where p is the proportion of male offspring in the sample and N_i is the number of families with i children. The sum S was used rather than single probabilities since there is a correlation between the size of a family and the sex of the older children.

The author confirms Winston's finding that (a) the sex ratio of upper class families is significantly above average and (b) as the size of the families of men married before 1910 increases, the sex ratio of the offspring approaches the sex ratio of the total population.

However, in the families of men married since 1918 any effect of family size and birth order on the sex ratio was found to have disappeared. There is a continuous rise in the sex ratio of upper class families in the last thirty years, more pronounced in large families. Today the sex ratio of both small and large upper class families in America and Germany is about 125 to 130 males for every 100 females.

Dividing the children of a group of 880 upper class German families into three groups according to whether they were born before, during, or after World War I, a steady rise in the sex ratio was found with time. The sex ratio of 1080 children born in Germany during World War I was higher than that of their prewar and lower than that of their postwar siblings.

81 JOSEPH BERKSON, M. D. (Division of Biometry and Medical Statistics, Mayo Clinic, Rochester, Minnesota). **Are There Two Regressions?**

Let u and v represent the true values of linearly related measures, d and e their respective unbiassed errors; $x = u + d$; $y = v + e$.

Two types of readings are distinguished: (1) an *uncontrolled* observation, (2) a *controlled* observation. An *uncontrolled* observation is made when wishing to ascertain the value of an unknown true quantity u_i, we measure the quantity. Example: the weighing of some given material with a chemical balance. A *controlled* observation is made when we wish to bring the quantity to a value u_i. Example: the weighing out of a *specified* amount in a chemical experiment.

For the uncontrolled observation

$$\bar{x} \text{ (from } n \text{ observations, with } u = u_i) \to u_i \qquad n \to \infty$$

For the controlled observation

$$\bar{u} \text{ (from } n \text{ observations, with } x = x_i) \to x_i \qquad n \to \infty$$

Two situations are contrasted: (1) there is an existent population, as for instance a bivariate normal distribution, from which samples are drawn; (2) there is no existent population, but observations are brought into being by a controlled experiment. The values of one variate, for example u (or x), are brought to specified readings u_i (or x_i) as controlled observations, and the corresponding value of the other variate is read as an uncontrolled observation y_i (or v_i). Examples: the bio-assay

experiment in which the dosages are controlled observations; an experiment to determine the electrical resistance of a circuit by adjusting volts to specified values and reading corresponding values of current, or adjusting current and reading corresponding values of volts.

In the situation of the existent population there are two regressions: the regression of v on u, and the regression of u on v. From a random sample or a sample selected on u with corresponding observation of y, the regression of v on u is estimated by minimizing the squared residual of the dependent variate y, but if u is measured with error as x the estimated regression is biassed, the more so the larger the error of x. Similarly the regression of u on v can be estimated if the independent variate is measured without error, but not if v is measured with error as y.

In the situation of the controlled experiment on u, with uncontrolled observation of the corresponding values y, minimization of the squared residual of the dependent variate y gives an estimate of the underlying regression. If the experiment is controlled on v with uncontrolled observation of the corresponding values of x, minimization of the squared residual of the dependent variate x yields an estimate of the same regression, that is, *there is only one regression*. Moreover if the measure of the independent variate is read with error as x (or y), the estimated regression is not biassed thereby, even though only the squared residual of the independent variate is minimized.

Certain statistical procedures and concepts derived from the model of sampling from an existent population do not apply to the situation of the controlled experiment. Examples are (a) the linear correlation coefficient, (b) test of linearity by analysis of variance, (c) judgment of "heterogeneity" of the data by the X^2 test.

82 REIERSOL, OLAV. (Purdue University). **The Identifiability of a Linear Relationship between Variables which are Subject to Error.**

Let x_1 and x_2 be observed variables, let y_1 and y_2 be the "true" values of these variables and let the errors v_1 and v_2 be defined by $v_i = x_i - y_i$, $i = 1, 2$.

We suppose that there exists an exact linear relation between the true variables $y_2 = \alpha + \beta y_1$, that the errors are independent of the true variables, and that the errors are independent of each other.

A parameter is said to be identifiable if it can be determined uniquely from the joint probability distribution of the observed variables. We

shall give necessary and sufficient conditions for the identifiability of the parameter β.

I. x_1 and x_2 stochastically independent:
   β not identifiable

II. x_1 and x_2 stochastically dependent.
   A. y_1 not normally distributed:
      β identifiable
   B. y_1 normally distributed.
      (1) Neither the distribution of v_1 nor the distribution of v_2 divisible by a normal distribution:
          β identifiable.
      (2) Either the distribution of v_1 or the distribution of v_2 divisible by a normal distribution:
          β not identifiable.

THE BIOMETRIC SOCIETY

Plans for the Second International Biometric Conference, to be held in Geneva, Switzerland, August 30-September 2, go forward. The preliminary program provides seven scientific sessions on biometry in relation to genetics, teaching and education, experimental design, its present status, industrial applications and biological assay, ending with a session of contributed papers. The Conference Committee has obtained accommodations for members from 7-9 Swiss francs and up. To insure reservations, members planning to attend the Conference are urged to send their requests to Professor Arthur Linder, 24 Avenue de Champel, Geneva, Switzerland, before May 20.

Dr. Eric C. Wood of England urges strongly that a start be made toward international standardization of biometrical nomenclature and symbolism. We quote from Dr. Wood's letter:

"Sirs,

I have thought for some time that it would be a good thing if biometricians and statisticians could arrive at some sort of international agreement about the nomenclature and symbolism they employ. This is not the place to enlarge on this topic, but I may perhaps give two examples selected from many that have occurred to me or have been mentioned by others.

"(1) The terms 'standard error' and 'standard deviation' are at the moment used interchangeably by many people. Some attempts have been made to restrict the term 'standard deviation' (and the symbol σ) to that parameter of the population which is estimated by the 'standard error' (with the symbol s) of the sample. Whether this is a good idea or not, it should be possible to make two such terms do two distinct jobs.

"(2) When the mean result of an experiment is quoted in the form 10 ± 1.0, what is to be understood by the quantity following the plus or minus sign? It should in my opinion be the standard error attaching to the result quoted. It should not be the standard error of a single one of the observations from which the mean result was calculated, and it should certainly not be the *probable* error, a quantity which might well disappear. Others may not agree with this opinion, but the figures in question should clearly have a unique meaning.

"I am aware that there are difficulties in achieving a large measure of standardisation at present, but it might be possible for a start to be made in certain restricted fields. I therefore suggest that the subject be placed on the programme for the next International Conference, and in the meantime the Society might form a Committee to enquire whether standardisation of the nomenclature and symbolism of statistics is desirable at all, and if so, to what extent.

"I may add that I have discussed this matter with the Officers and other Committee-Members of the British Region, and while they have not seen this letter and must not be taken necessarily to agree with its wording, they are in agreement with the general principle that the matter is worth consideration.

I am, Sirs,

Yours faithfully,

ERIC C. WOOD"

As general officers of the Society for 1949 the Council has re-elected President R. A. Fisher, Treasurer J. W. Hopkins and Secretary C. I. Bliss. Because of the long time required for ballots to reach members of the Australasian Region, the election of new Council members will be reported in the next issue.

We are happy to report the formation of two new regions of the Society. The Indian Region was organized in Allahabad during the sessions of the Indian Science Congress Association in the first week of January, under the chairmanship of P. C. Mahalanobis, Vice-President of the Region. The meeting followed an active campaign to enroll new members. Professor M. Frechet, Faculté des Sciences de Paris, and Dr. D. Schwartz, Service des Recherches Biologiques du S.E.I.T.A., 2 Ave. d'Orsay, Paris 7é, have been named as provisional Vice-President and Secretary-Treasurer, respectively, of the French Region. Following an enrollment of new members the first meeting of the Region was scheduled for February or March. The proposal of a joint French-Italian Region was still under consideration when this issue of *Biometrics* went to press.

The Australasian Region, still in its infancy in the last issue of this journal, is now fully organized, with thirty-seven members, as of December, 1948, from Victoria, New South Wales, Queensland, South Australia and New Zealand. Dr. E. A. Cornish of the Council for Scientific and Industrial Research at the University of Adelaide is Vice-President, and Dr. Helen N. Turner of the McMaster Animal Health Laboratory in Glebe, N.S.W., is Secretary-Treasurer. The first meeting of the Region as a whole was held in Melbourne on January 8th with 30 or more in attendance from New South Wales, South Australia, Victoria and Australian Capital Territory. The Victorian Branch of the Region has been in active operation since last summer, with program sessions in August, October and November. Bi-monthly meetings of the Victorian Branch are planned for 1949.

The Western North American Region met jointly with the Institute

of Mathematical Statistics in Seattle at the University of Washington, on November 27. Two sessions were devoted to fishery biology, with papers in the morning by W. S. Rich, J. Neyman, R. Silliman, D. Chapman and E. L. Scott, W. F. Thompson presiding, and in the afternoon by O. E. Sette, S. C. Dodd and M. E. Schaefer, F. W. Weymouth in the chair. Regional by-laws were adopted for confirmation by the Council.

The annual meeting of the Eastern North American Region was held in Cleveland, Ohio, on December 27–30, in conjunction with the Biometrics Section of the ASA, the Institute of Mathematical Statistics and the American Public Health Association. The following regional officers were named for 1949 at the business meeting on December 30: C. P. Winsor, continuing as Vice-President; Roland H. Noel, Secretary-Treasurer; Lloyd C. Miller and Paul T. Bruyere, members of the Regional Committee for the term 1949–1951. The scientific program comprised seven sessions.

> December 27. *Symposium on Statistics for the Clinician—Clinical Problems*, with Joseph Zubin as chairman and papers by A. L. Baldwin, J. F. Kubis, L. S. Kogan and J. McV. Hunt, A. I. Rabin, and L. J. Cronbach. A *Panel Discussion of Medical Statistics* with J. A. Rafferty as moderator and topics presented by D. F. Votaw, Jr., N. Scrimshaw, J. Neyman and C. W. Heath, was followed immediately by a *Symposium on Statistics for the Clinician—Proposed Solutions*, with J. Zubin as chairman and papers by E. F. Gardner, D. Horn, H. Guetzkow, and G. W. Brown.

> December 28. A *Round Table on Morbidity Statistics* with H. Muench as chairman included the following discussants: T. D. Woolsey, F. Moore, S. D. Collins, B. S. Sanders, and W. T. Fales. A session on *Bioassay* had H. C. Fryer as chairman and papers by C. I. Bliss, N. Turner and D. F. Votaw, Jr., O. Kempthorne, M. W. Greene and L. F. Knudsen, and J. Ipsen.

> December 29. *Effects of Errors in the Independent Variate in Regression Problems* under the chairmanship of W. E. Deming offered papers by J. Berkson, O. Reiersol and J. Neyman.

The first meeting of ENAR in 1949 was a two-day conference on *The Place of Statistical Methods in Biological and Chemical Experimentation*, at the American Museum of Natural History, New York City, on January 28 and 29. The conference was sponsored jointly with the New York Academy of Sciences and the New York Metropolitan Chap-

ter of the American Statistical Association. Papers were presented by G. W. Snedecor, G. M. Cox, F. Wilcoxon, W. J. Youden, K. A. Brownlee, R. A. Harte, C. V. Winder, H. C. Batson, C. I. Bliss, L. F. Knudsen, L. C. Miller, B. J. Vox, D. Mainland, D. D. Reid, H. M. C. Luykx, and F. E. Linder.

The Western North American Region will co-sponsor a session on June 17 at 9:00 A.M. on the application of statistics to biology, at the University of California in Berkeley. This forms part of the Fifth Regional West Coast Meeting of the Institute of Mathematical Statistics.

The Biometric Society will publish its first Directory in the spring of 1949, and each member will receive a copy gratis. It will include a list of all members through April 1949, and information concerning the organized regions, their officers, activities and by-laws.

The Society has a new home in New Haven at 321 Congress Avenue, in space provided through the kindness of the Department of Public Health of Yale University. Mail for the secretary should still be addressed to Box 1106, New Haven 4, Connecticut. Mrs. Elizabeth G. Weinman has joined the Society as Executive Assistant to the Secretary. A Standard Duplicating Machine has recently been added to the office equipment and addressograph plates have been made for all members. It is hoped that this will facilitate the speedy and efficient distribution of information to members.

NEWS AND NOTES

Special Summer Session in Survey Research Techniques. The Survey Research Center of the University of Michigan will hold its special summer session in Survey Research Techniques from July 18 to August 13, 1949.

The following courses will be offered: Introduction to Survey Research, Survey Research Methods, Sampling Methods in Survey Research (elementary and advanced), Mathematics of Sampling, Statistical Methods in Survey Research, Techniques of Scaling.

In addition the introductory courses will be given from June 20 to July 16. This will permit students who are attending the full eight-week summer session of the University (June 20 to August 13) to register for the introductory courses during the second four weeks.

It is expected that this special session will attract men and women employed in market research or other statistical work and university instructors and graduate students with a particular interest in this area of social science research.

All courses are offered for graduate credit and students must be admitted by the Graduate School. Inquiries should be addressed to the Survey Research Center, University of Michigan, Ann Arbor, Michigan.

Statistics Summer Session in Applied and Mathematical Statistics. The Institute of Statistics of The University of North Carolina announces a statistics summer session, June 9 to July 19, 1949 at Chapel Hill. Intensive statistical instruction will be offered for the benefit of (1) research scholars in other sciences who want a practical working knowledge of statistical theory, (2) statistical consultants in various fields, (3) those preparing to teach statistics or to develop statistical theory, and (4) students working toward a degree in applied or theoretical statistics.

The instructional staff consists of the following professors: G. W. Snedecor, for fifteen years Director of the Statistical Laboratory at Iowa State College and author of the widely used textbook "Statistical Methods"; D. J. Finney, Lecturer in the Design and Analysis of Scientific Experiment, University of Oxford, England; J. Wolfowitz, Associate Professor, Department of Mathematical Statistics, Columbia University; and three members of the staff of the Institute of Statistics, R. C. Bose, Professor, recently from Calcutta University, India; Herbert Robbins, Associate Professor, and Gertrude M. Cox, Director.

An announcement of this summer session may be secured by writing to Director, Institute of Statistics, The University of North Carolina, Box 168, Chapel Hill, North Carolina.

STATISTICS IN MEDICINE—J. P. GRAY, M.D., Parke, Davis and Company. "The need for statistical method in medicine extends as the horizons of medicine itself are extended. The use of quantitative method has invaded each special field within medicine, perhaps to the greatest extent in the broadest field . . . public health . . . in which it is indispensable.

"There are those who attempt the practice of medicine on statistical bases, but, fortunately for patients, these are few. Many physicians are aware of inadequate background on attempting analysis and interpretation of data taken from clinical records involving few or many patients. From other physicians, however, unaware of such inadequacies, come evidences thereof, in spite of qualified editors of acceptable journals, for sometimes their writings are published. One recurring example, irritating to statistician and statistically minded reader interested in specificity and accuracy of definition of classification and other details, is found in the use of the term 'mortality rate' in referring to a number expressed per centum. Perhaps usage will be successful in changing basic definitions, but how can this path be justified when specific definitions already exist? It is NOT 'more difficult' or 'more technical' to be accurate and to use the proper term for the rate referred to—'case fatality rate.' Yet this inaccuracy continues, almost unbounded, in current medical literature.

"Undergraduate students of medicine logically might be expected to be interested, necessarily if not inherently, in quantitative method; but they frequently manifest unmistakable disinterest in statistics and statistical method, per se. If the interested teacher substitutes flank movement for frontal attack, interest of advanced undergraduate or graduate students of medicine can be aroused, probably because they have had opportunity to develop appreciation of need for analysis and appraisal of groups of observations.

"Further, an appeal usually successful involves teaching which utilizes current or recent literature in which the author has demonstrated inadequacy in soundness of presentation, or in analysis and interpretation of data. In such instances, little emphasis is required to indicate the hazard to which the author has subjected himself through his inadvertent invitation to embarrassment, at least in the estimation of the critical

reader (and physicians and undergraduate students of medicine pride themselves on membership in this category!) who, applying basis tests of statistical significance, finds that data presented do not justify conclusions drawn.

"Research workers, in medicine and in the medical sciences, comprising but a relatively small group, appreciate the importance of statistical method as a basic technique, comparable to language for transmitting thoughts and ideas to others, not to discount its use in planning.

"Statistics, therefore, has taken its place as a basic requirement in adequate preparation of the student of medicine, combining logic, mathematics, and a guiding philosophy applicable to experimental, laboratory, and clinical aspects of medicine. Admittedly, statistics has its limitations and its pitfalls, but the intelligent, honest, sound use of quantitative method will save the student, the investigator, the clinician, from pitfalls even more hazardous to be encountered by the worker oblivious to or unappreciative of the method and its usefulness."

TEACHING OF STATISTICS TO PUBLIC HEALTH WORKERS—BEN FREEDMAN, M.D., Director, Training Center, Department of Health, New Orleans, Louisiana. "The attitude of medical men towards statistics is varied. Those who realize it is an instrument for understanding events have respect for biometrics; many who do not have this realization believe in the contrite dictum that statistics can be made to corroborate anything. I believe that all public health personnel should at least know the value of the statistical method even though they may not know the detailed technique in using it. I do believe that post-graduate schools in public health are overdoing the teaching of statistics to health officers. There should be several levels of statistical courses in such schools, ranging from statistical appreciation to the detailed study of statistical method.

"I will agree that the more one knows about the techniques of a discipline the more one will appreciate the discipline, but I also believe that it is possible to teach the appreciation of a subject like statistics without subjecting the half-interested health officer to the rigors of a course in technique. Of course, those who have a background in mathematics will find the so-called rigorous course quite simple. There is too much about public health administration to be learned and to be taught in a school of public health so far as the ordinary practical health officer is concerned than worrying for six months whether he is going to pass a course in mathematics.

"Let me repeat, I believe that every public health worker should know the significance of statistics as far as it is possible, but the teaching of statistics should bear more relation to the background of the individual, to the work he is going to do, and to all remaining subjects that he has to learn at the same time."

ENGLAND—**J. Moyal,** has joined the Mathematical Statistics staff at the University, Manchester. He and **Maurice Bartlett** are collaborating on a book on Stochastic Processes . . . **Maurice G. Kendall,** Statistician and Joint Assistant Geneial Manager to the Chamber of Shipping, "The new member of the family has arrived, named James. His growth curve, which I am plotting carefully from week to week, has for the first five weeks shown the unusual feature of sloping away from the time axis instead of towards it . . . Obviously this cannot continue and the curve will have to turn itself into a logistic sooner or later; but at the moment the rate of increase is more than satisfactory." . . . **K. A. Brownlee,** formerly with The Distillers Company, Ltd., Great Burgh, Epsom, Surrey is now with E. R. Squibb and Sons, New Brunswick, New Jersey. He reported on a confounded fractionally replicated experiment in penicillin production at a conference of The Biometric Society and Section of Biology of The New York Academy of Sciences (New York, January 28).

UNITED STATES—Reports have been received that **R. E. Blaser,** Department of Agronomy, Cornell University is back at the office carrying a normal load. We hope your luck has changed, no more "hits" . . . **R. J. Borden,** Hawaiian Sugar Planter's Association, Honolulu, sends a description, a part of which can be quoted. "We have just finished our annual year-end series of meetings with sugar plantation technologists and executives. **L. D. Baver** kept things going smoothly, and interjected enough of his well-pointed stories to keep the groups in good humor. He certainly does have the art of pulling the right story out of the bag at the right moment." Mr. Baver is Director of the Experiment Station of the Hawaiian Sugai Planters' Association . . . **H. L. Bush,** The Great Western Sugar Company, Longmont, Colorado, states, "We are still employing lattice designs in our variety testing programs and in some other tests. The triple lattices seem to be the best adapted to our work." . . . **M. Lois Calhoun,** is now Head of the Department of Anatomy, School of Veterinary Medicine, Michigan State College, East Lansing. She has a new DeSoto and has ideas for the growth of the Department . . . **J. H. Curtiss,** Chief, National Applied Mathematics

Laboratories has appointed himself as Acting Chief of the Section, Institute for Numerical Analysis, Los Angeles. Some readers may be interested in a portion of a letter from Mr. Curtiss. "A good deal of the work in our Statistical Engineering Laboratory deals with the application of modern statistical methods to research in physics and chemistry, and not just to industrial research. This is a virgin field for Fisherian statistics." . . . **Mary L. Dodds,** Acting Head, Foods and Nutrition Department, The Pennsylvania State College, has expressed a belief that "word of mouth is necessary for establishing understanding between a biochemist and a statistician." . . . **Daniel R. Embody**, who left Cornell University in 1942 to join the United States Navy, Bureau of Ships is now Director, Embody Statistical Laboratory, Spirit Lake, Idaho. He has a consulting and calculating business . . . **Joseph F. Pechanec,** Chief, Division of Range Research, Forest Service, Portland, writes, "The thin veneer of statistics I acquired at Iowa State has become badly eroded during the past ten years by the grind of supervisory and other activities . . . Your excellent publication, "Biometrics" brings on a cold sweat. Nevertheless, I do appreciate fully the vital need for and the extreme usefulness of tools provided by statistical methods and experimental design. To that extent, at least, my statistical training has been of immeasurable value." . . . **George Snedecor,** past-president of the American Statistical Association addressed the Central Indiana Chapter of the American Statistical Association at a dinner meeting at Purdue University, Lafayette, Indiana, on Tuesday November ninth. There were about 75 persons present, many coming from Indianapolis. The subject was "On The Design of Sampling Experiments." The same afternoon he spoke to 300 students and faculty of the university on "Selected Topics in Design of Experiments and Samplings." He was a guest of **Carl F. Kossach**, director of the newly formed statistical laboratory at Purdue University.